

Application of Markov Chain in Forecasting Demand of Trading Company

Hamed Alipoor Talemi, Kiyoumars Jahanbani, Arash Heidarkhani,

Afshin Azad Khomami, Amin Torabi Gol Sefidi, Seyed Abbas Abolghasemi

^{1,2,3,4,5,6} M.A. Student of Business Management, Rasht Branch, Islamic Azad University, Rasht, Iran

Abstract

Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making (industrial and commercial) use it. Markov processes arise in probability and statistics in one of two ways. Markov process is a tool to predict that it can be make logical and accurate decisions about various aspects of management in the future. A stochastic process, defined via a separate argument, may be shown mathematically to have the Markov property, and as a consequence to have the properties that can be deduced from this for all Markov processes.

Keywords: Markov Chain, Forecasting Demand, Trading Company

1. Introduction

Management is defined decisions in a simple form and the most important factor for decision making is forecasting future. At present era those organizations have high complexity and much information is available, so their arrangement and refine can help to management in a logical and accurate decisions. The use of various aspects of operations research is easier to deal with complex issues for managers. Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making (industrial and commercial) use it. Successful decision is a picture of the future that this will not be achieved only from the prediction, based on scientific principles. Markov process is a chain of random events that by having information about the current location can be predicted next period and in fact Markov chain is a tool that used for forecasting of situation organization in future periods.

2. Definition of Stochastic Processes and Markov Chains

A random process is defined as a set of random variables $\{x_t\}$ that defects of T is often referred to time, there is around a specific set of

T (hakimipour, 1998). If random process $\{x(t), t \in T\}$ be such that with identify the value of $x(s)$ values of $x(t)$ for $t > s$ depends on the values $x(u)$ make for $u < s$, then its process is called Markov process that such a process is defined by the following expression:

If for $t_1 < t_2 < \dots < t_n < t$

$$P_r \{a \leq x(t) \leq b | x(t_1) = x_1, \dots, x(t_n) = x_n\} =$$

$$= P_r \{a \leq x(t) \leq b | x(t_n) = x_n\}$$

It is result that process $\{x(t), t \in T\}$ is Markov process (soldo, 2012).

Markov analysis began with the research. A. Markov, Russian theorist in years (1907 - 1906) in the field of probability science. However, a comprehensive theory processes and analysis of Markov by. A. n. gorov - w. dublin - p. loe - j. l. dub & others were completed it in 1930 (Hakimipour, 1998).

Markov chain is certain models from more general potential model that these models are famous random processes and in them current state of a system depends on all previous states. Markov process is a random process. With this feature that current state of system is dependent just prior final state of the system. In other words in prediction of state of a system at the future times according to the information related to current state of system will be analyzed (Wang, 2004). In fact Markov analysis, it will establish a way to the current analyze for certain variables because is possible to predict the future movement same as variables because Markov process is chain of a random events that by having the current success can be predicted the next phase (Lee, Tong, 2011).

The field of Markov chains has been widely used in management science that these can include:

1. Human resources planning model
2. Pyramid Maslow's model on human needs
3. Model to predict price changes.
4. Changes in brand by customer product
5. Behavior of customer receivables

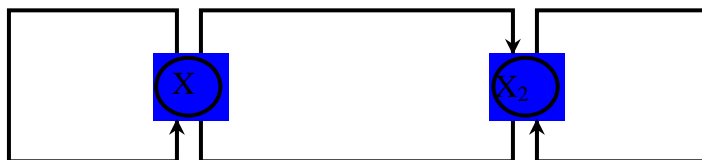
- 6. Maintenance model
- 7. Inventory control
- 8. Describing a particular type of storage issues
- 9. Analysis and replacement human resources
- 10. Prediction of system reliability
- 11.

3. Basic Concepts of Markov Processes

For analysis of system from Markov model should determine the two basic elements of the system and the possibilities of movement between them. System state is the status of the system at a moment in time like a car works or does not work at a moment in time. Probability of movement among the states call transition Probabilities that representing the system moves from one state to another state during a specified period ($i \rightarrow j$) Probability of some change from one state to another state in the next period is called transition Probabilities of the Markov model $P\{X_{t+1}=j|X_t=i\}$
 If the number of possible states in Markov chain is limited, transition probabilities can show to form a matrix that this matrix in Markov process will be always square ($m * m$). In this matrix P_{ij} show transition probabilities from state i to state j in next time period (Deng, 1990).

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & & j & & m \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ \vdots \\ i \\ \vdots \\ \vdots \\ m \end{matrix} & \left[\begin{array}{cccccc} P_{11} & P_{12} & \dots & \dots & P_{1j} & \dots & \dots & P_{1m} \\ P_{21} & P_{22} & \dots & \dots & P_{2j} & \dots & \dots & P_{2m} \\ \vdots & \vdots & & & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots & & & \vdots \\ P_{i1} & P_{i2} & \dots & \dots & P_{ij} & \dots & \dots & P_{im} \\ \vdots & \vdots & & & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots & & & \vdots \\ P_{m1} & P_{m2} & \dots & \dots & P_{mj} & \dots & \dots & P_{mm} \end{array} \right] \end{matrix}$$

Also transition probabilities can show in form of graph.



Every issue to resolve by a Markov process should be having the following three characteristics:

- 1- Transition probabilities are dependent on only the current state of the system.
- 2- Transition probabilities are always fixed.
- 3- Sum of transition probabilities move to other states in the next time period should be equal one.

4. Prediction of Future States

Predictions of future states require that initial states of the system are identified and transition probabilities fixed. There are several methods for predicting future states that in this paper refers to as case to the matrix multiplication (Yamaguchi, Nagai, 2006). Matrix multiplication is a simple method for predicting the state of a Markov system for future periods. By having the initial state of matrix multiplication can be used for prediction system at time.

A) The state of the system:

First system state at time n is shown by a one-dimensional matrix to name of vector

$$P(n) = \{P_1(n), P_2(n)\}$$

That in this relation $P(n) =$ value vector $(n) \cdot P_1$

$P_1(n) =$ the probability that system at time n be in state 1

$P_2(n) =$ the probability that system at time n be in state 2

If we suppose that system at time n be in state 1 Then $P_1(n) = [1, 0]$

If we suppose that system at time n be in state 2 Then $P_2(n) = [0, 1]$

It is necessary to note here that if there is a system of more than two states, not necessary that the system in the initial state is only one of the states and may be more than one state but in any state vector sum must always be equal to one. For example, the state vector for a system of three cases may be $[0/1 \text{ and } 0/7 \text{ and } 0/2]$

b) Matrix of transition probabilities (p):

Transition probabilities matrix is shown as following

$$P = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{matrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{matrix} \end{matrix}$$

In this matrix:

$P =$ Transition probabilities matrix

$P_{ij} =$ Transition probabilities of system from state i to state j

C) Prediction of the future state:

To calculate the probability that system at time $(n+1)$ be state j , we use as follows relationship

$$P(n+1) = P(n) \cdot P$$

That this relationship: $P(n+1) =$ State probability vector at time $n+1$ (one time period later)

$P(n) =$ State probability vector at time (n) (current time period)

$P =$ Transition probabilities matrix

According to above relationship can be calculated transition probabilities in several periods later in form of simple.

$$P(n+2) = P(n+1) \cdot P$$

$$P(n+3) = P(n+2) \cdot P$$

A general relationship is obtained from these relationships that as follow:

$$P(n) = P(0) \cdot P^n$$

That this equation:

$P(n) =$ State probability vector in time period n th

$P(0) =$ probability vector in time period zero (starting point of time that probability of each state is certain)

$P^n =$ power of n th in transition probabilities matrix

$n =$ Number of time periods for which it is predicted.

Therefore it can be done necessary projections of this relationship for any period of time and used for decision making in planning (Zhang, 2001).

5. Stable State Conditions

In Markov process often by more n (in long term) value vector tends to fixing state (Stable state). As to achieve its period multiplying the state vector in transition probabilities matrix is equal to transition probabilities matrix in periods later that this state is called Stable state.

If stable value show by π symbol, instable state, the state vector will be in terms of decimal values as follow:

$$\pi = [\pi_1, \pi_2]$$

In this relationship:

$\Pi =$ State probability vector (as relative amounts)

$\pi_1 =$ amount of state 1

$\pi_2 =$ amount of state 2

Since in stable state conditions isn't important time period and values are independent of time, multiplication of state vector in transition matrix a vector in stable state will be same as state vector. Therefore stable state values can be determined in the following manner algebra:

$$\pi = \pi.P \Rightarrow \pi = [\pi_1, \pi_2] \times \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \Rightarrow \begin{cases} \pi_1 = \pi_1.P_{11} + \pi_2.P_{21} \\ \pi_2 = \pi_1.P_{12} + \pi_2.P_{22} \end{cases}$$

And on the other hand sum of probability states must be equal to one: $\pi_1 + \pi_2 = 1$

Therefore we have three equations and two passive that solving it will obtain value of the state vector in period n that it is beginning of stable state. A Markov process may not reach always stable state. However, it can be stated that if there is a value of n that all elements of P^n be greater than zero, in this conditions there will be a Stable state. To calculate the Stable state can be used MicroManager Software that is applicable to in the analysis of Stable state (Ujjwal Kumar, Jain, 2010).

6. Conclusion:

Markov process is a tool to predict that it can be make logical and accurate decisions about various aspects of management in the future. Also Markov is random process. With these differences in Markov process to predict the future state is used information related to the last period that this is one of the analyzable fundamental principles through Markov chains.

In transition probability matrix that rows and columns should always be equal (it be square shape) sum of rows is equal to one.

In prediction of future states based on a Markov chain if prediction are calculated for several period consecutive of a system. Finally, after some time the results will be the same that is called stable state and this caused by interference only data of one period (zero period) in calculation.

References

- Hakimipour,A.(1998). decision making in Management, Astan Quds Razavi - Second Edition
- Soldo,Bozidar(2012). Forecasting natural gas consumption, Applied Energy,26-37.
- Wang,Chao-Hung (2004). Predicting tourism demand using fuzzy time series and hybrid grey theory ,tourism Management,367-374.
- Deng JL (1990) .Control Problems of Grey s\System,Wuhang: Huazhong University of Science and Technology Press.
- Yamaguchi,G.D.L i, D .Nagai,M. (2006) . A grey- Based Approach to suppliers selection problem,Proc.Int.Conf.onParallel,Distributed Processing Techniques and Applications.
- UjjwalKumar,V.K.Jain (2010) .Time series models(Grey-Markov,Grey Model with rolling mechanism and singular spectrum analysis) to forecast energy consumption in India,Energy,35:1709-1716.
- Yi-ShianLee, Lee-Ing Tong (2011). Forecasting energy consumption using a grey model improved by incorporating genetic programming,Energy Conversion and Management,52:147-152
- Zhang SJ,He Y (2001). A Grey – Markov forecasting model for forecasting the total power requirement of agricultural machinery in Shangxi Province. J Shanxi AgricUniv(Nat Sci Edi),21(3):299-302.