A Performance Study of an Implementation of the Push-Relabel Maximum Flow Algorithm in Apache Spark's GraphX

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Background Motivation

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- As a result, parallel programming technologies have been developed (e.g. MapReduce)
- Many algorithmic solutions to problems need to be revisited in parallel.

Apache Spark

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- Built in Scala, based on "Resilient Distributed Datasets", or RDDs

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- Data partitioning abstraction

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- Achieves fault-tolerance through lineage
- Allows caching of data between parallel operations

GraphX

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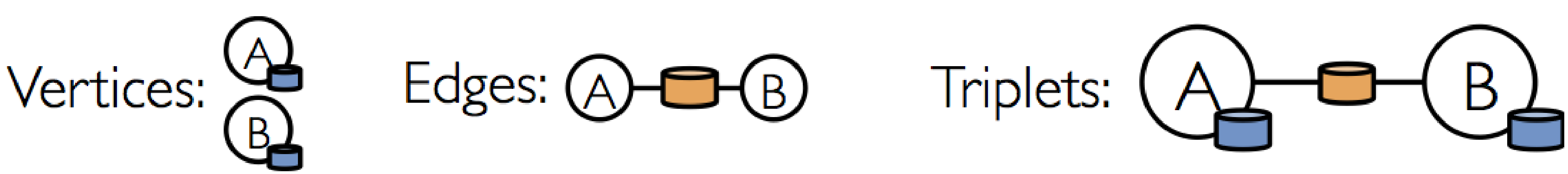

Simple GraphX Example

- What if we wanted to find the oldest follower of each person in the graph?

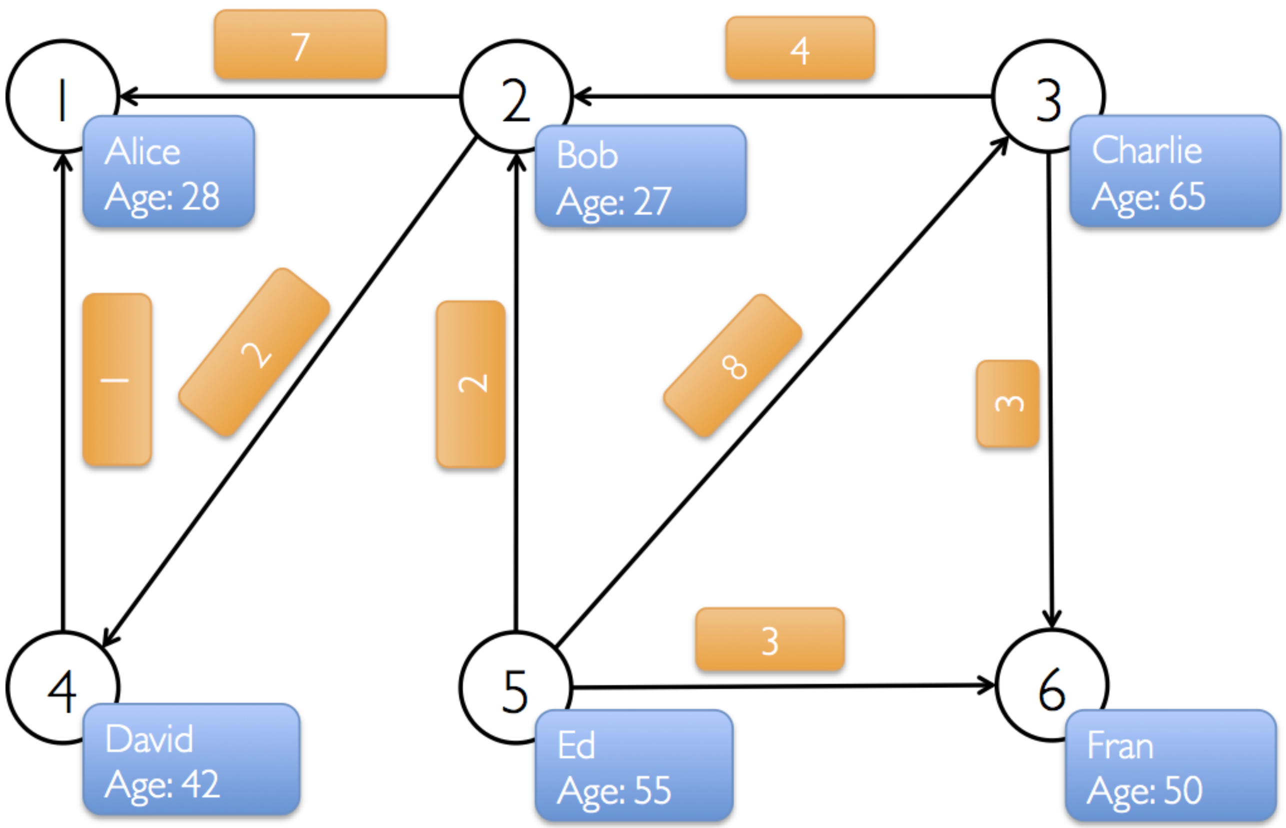

Image Source: http://ampcamp.berkeley.edu/big-data-mini-course/graph-analytics-with-graphx.html

Simple GraphX Example

```
// Find the oldest follower for each user
val oldestFollower: VertexRDD[(String, Int)] =
  userGraph.aggregateMessages[(String, Int)](
    // Map Function
    edge => edge.sentToDst((edge.srcAttr.name, edge.srcAttr.age)),

    // Reduce Function
    (a, b) => if (a._2 > b._2) a else b
  )
```

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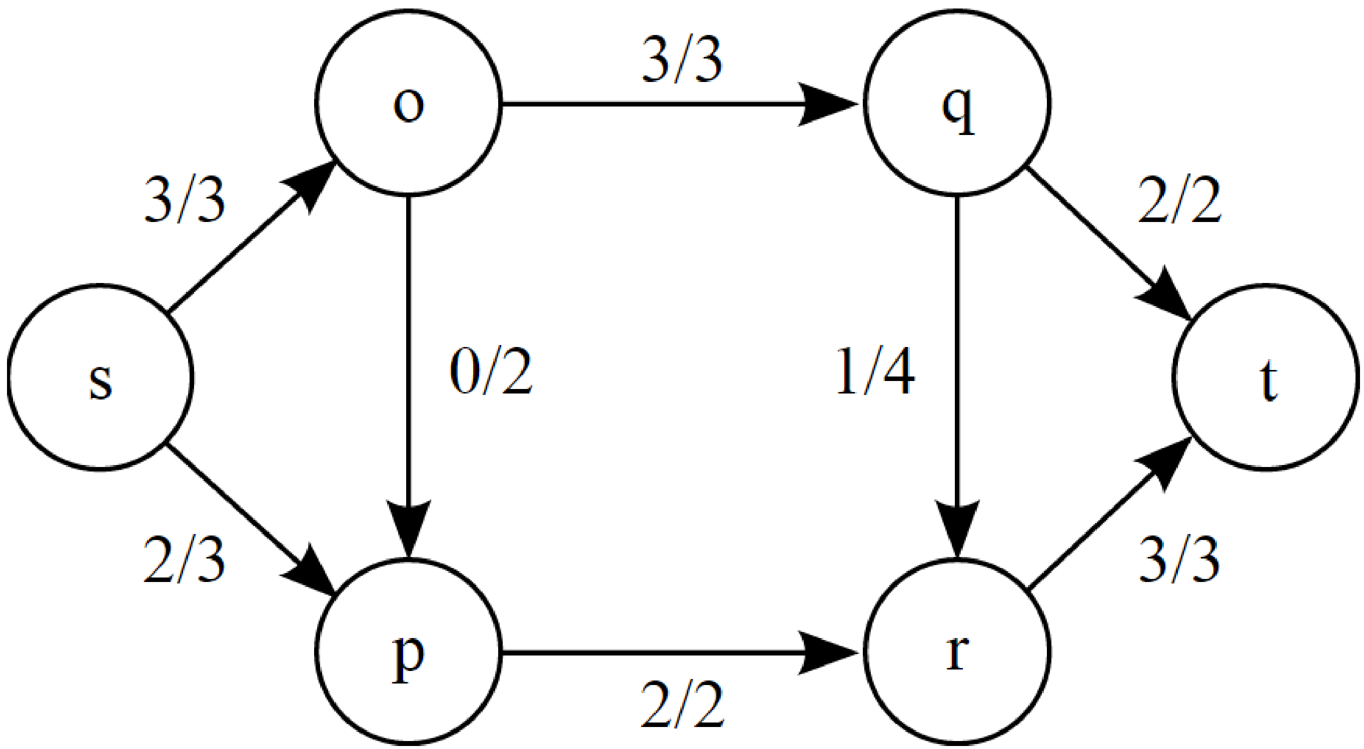
Maximum-Flow Problem



What is the maximum flow that can be pushed from the source vertex to the sink vertex?

Push-Relabel Algorithm

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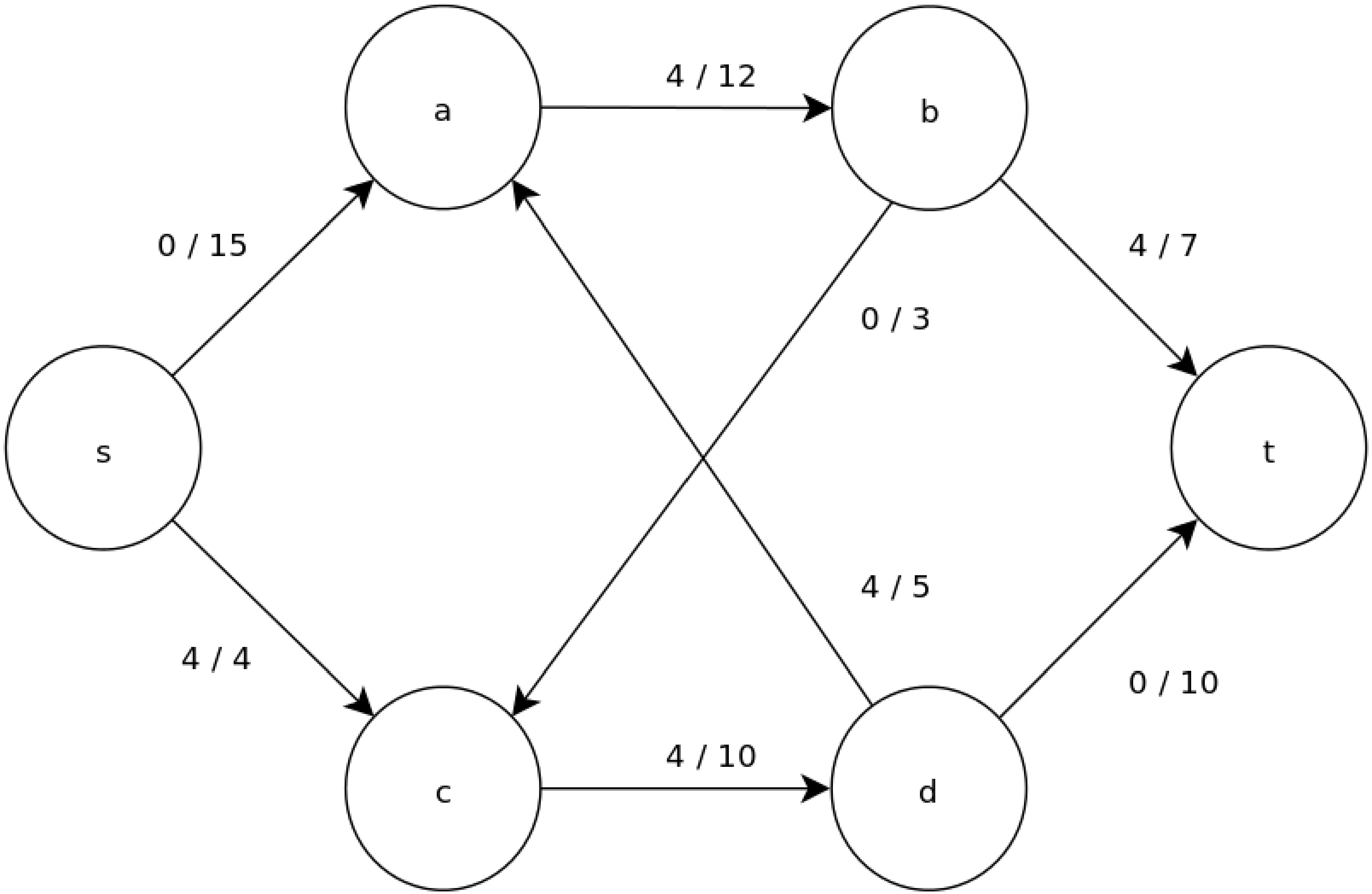
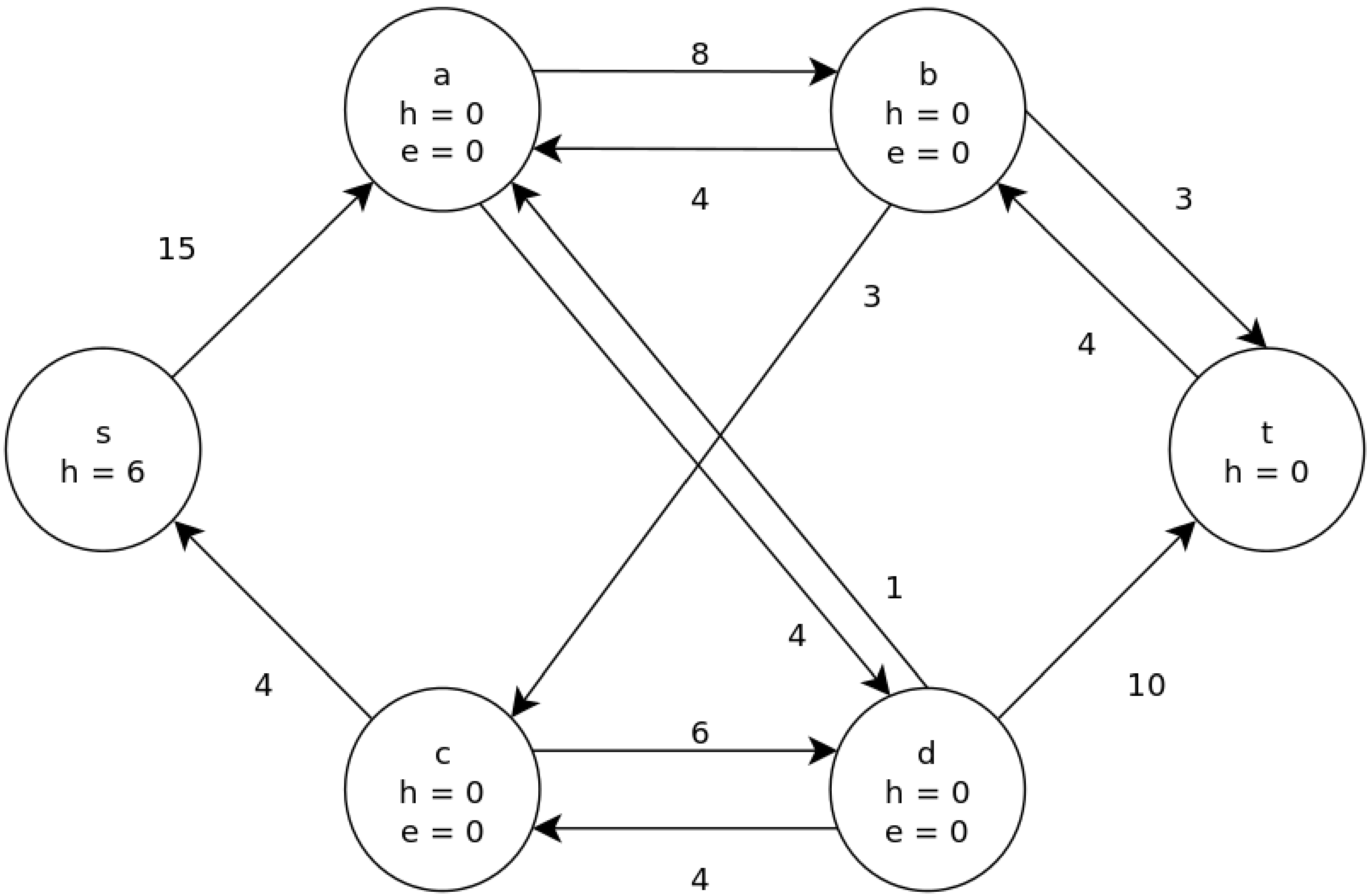
- Utilizes the concept of “preflow”
- Labeling mechanism monitors which vertices are eligible to push excess flow

Push-Relabel Example



Source: http://en.wikipedia.org/wiki/Push%E2%80%93relabel_maximum_flow_algorithm

Push-Relabel Example



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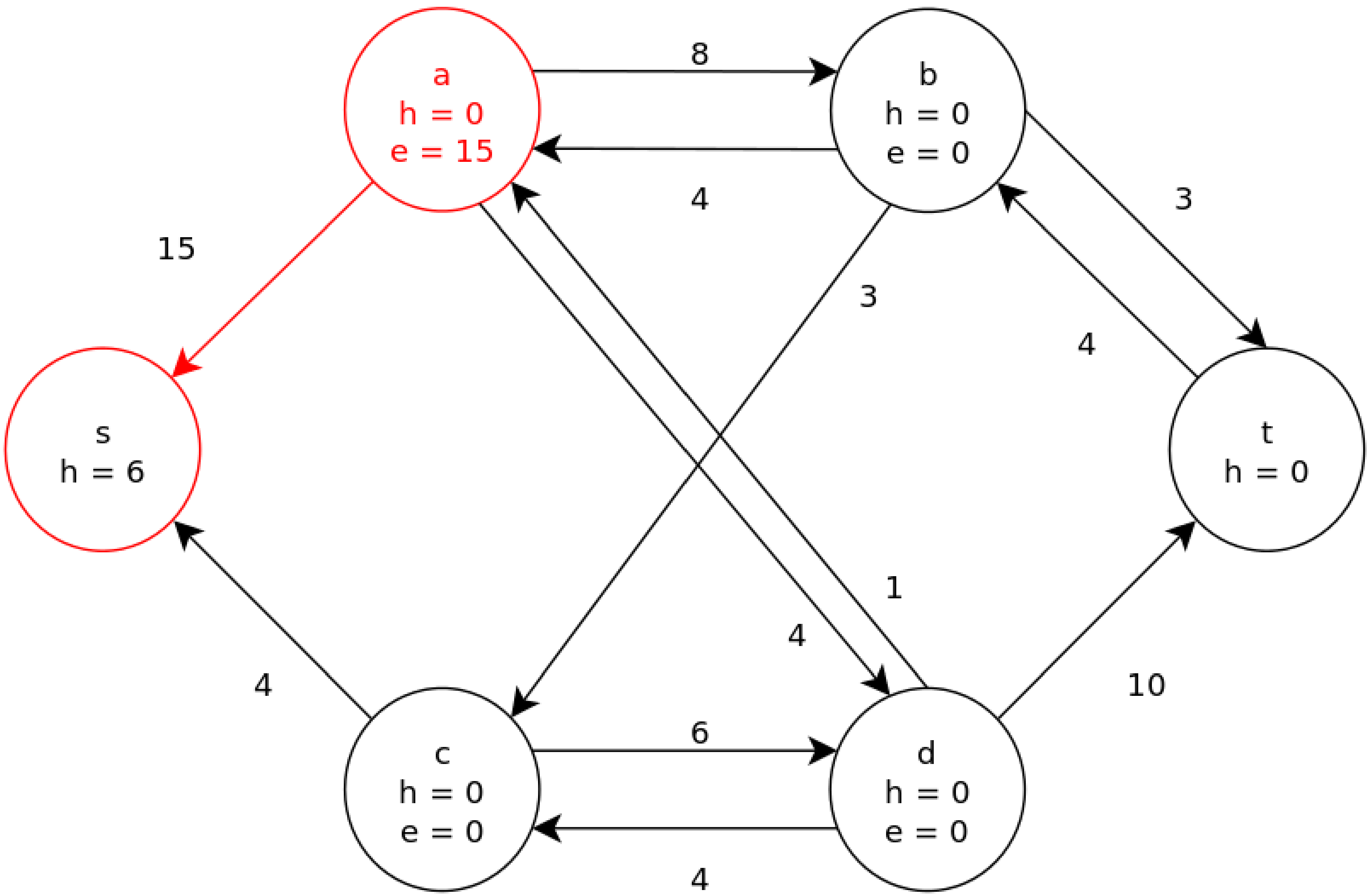

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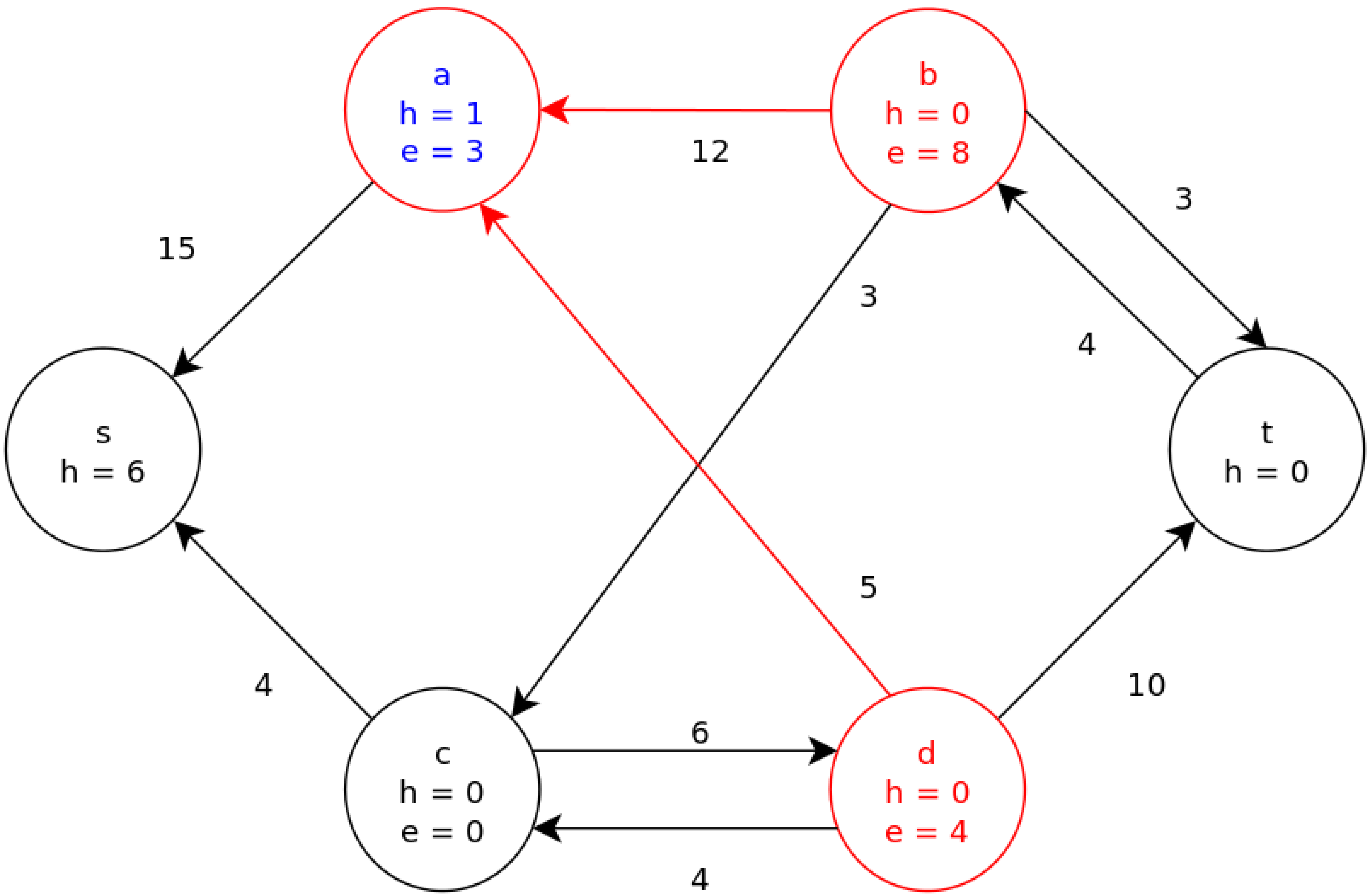
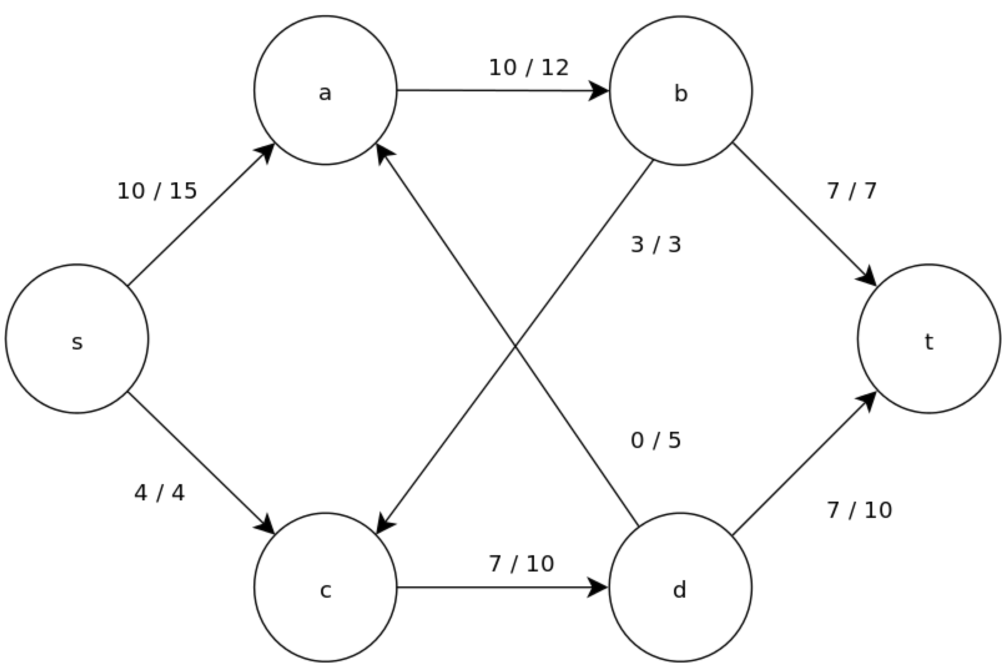


Project Goal

“Implement a solution to the maximum-flow problem in GraphX, targeting the Push-Relabel algorithm as our approach.”



GraphX Pregel API

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  - “Merge Message” function
  - “Vertex Program” function

GraphX Pregel API Consideration

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  - Use the **Send Message** step to find possible pushes or relabels in the graph.

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- Basic Approach
  - Use the **Send Message** step to find possible pushes or relabels in the graph.
  - Use the **Merge Message** step to choose which operations will be executed based on excess.
  - Use the **Vertex Program** step to update the values of the graph.

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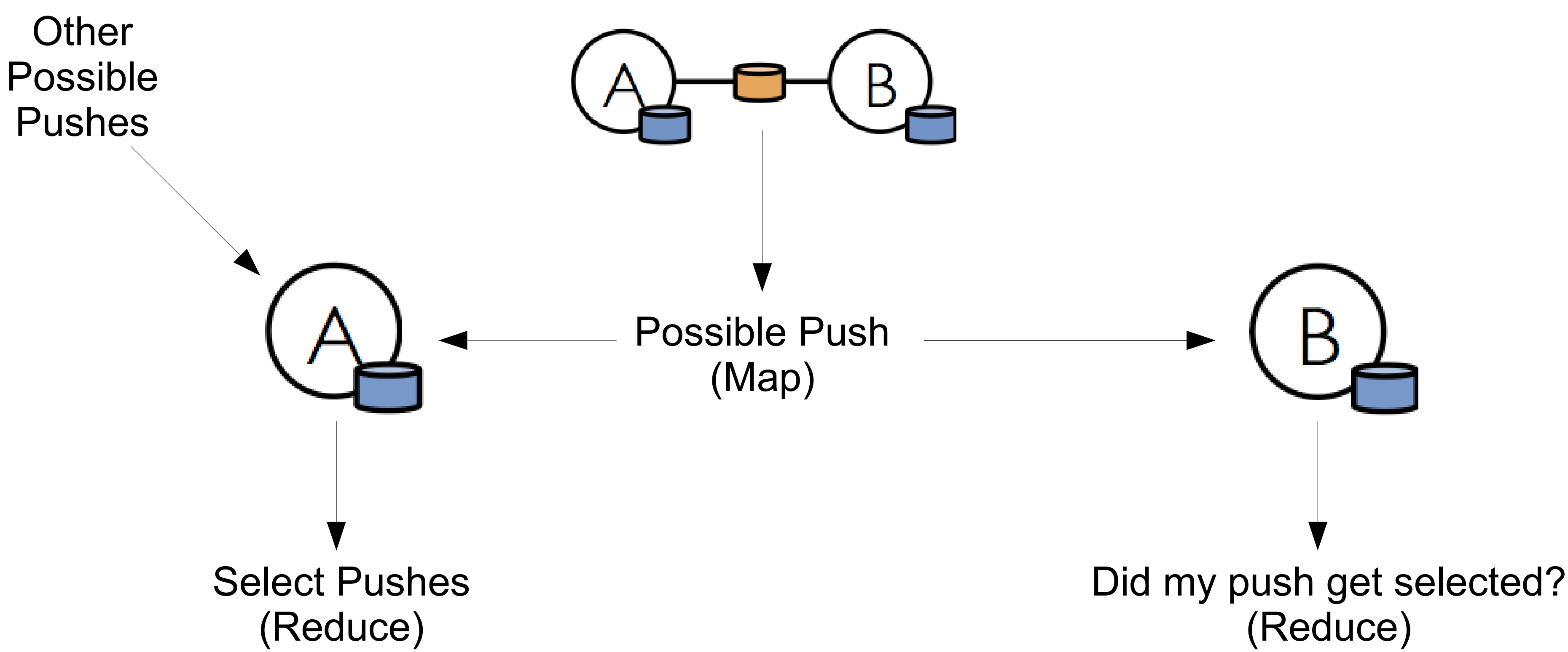
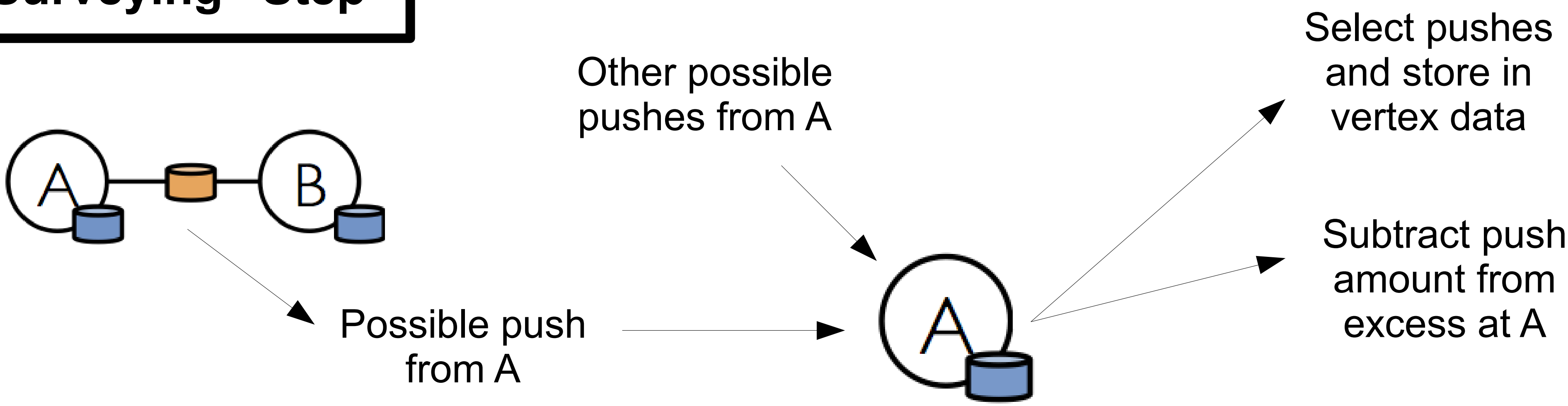
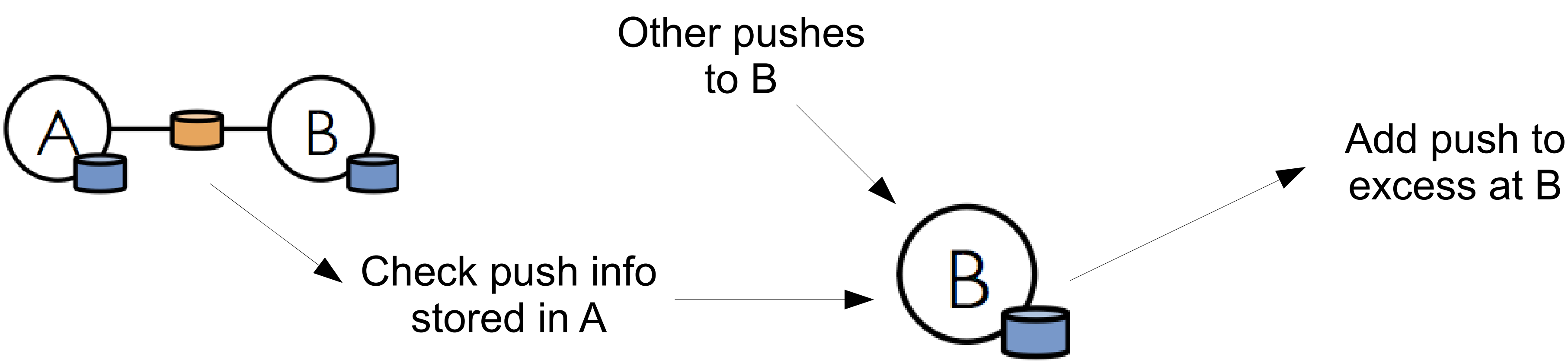


New Approach Visualization

“Surveying” Step



“Execution” Step



Handling Relabeling

- Relabel information also needs to be included in the messages.

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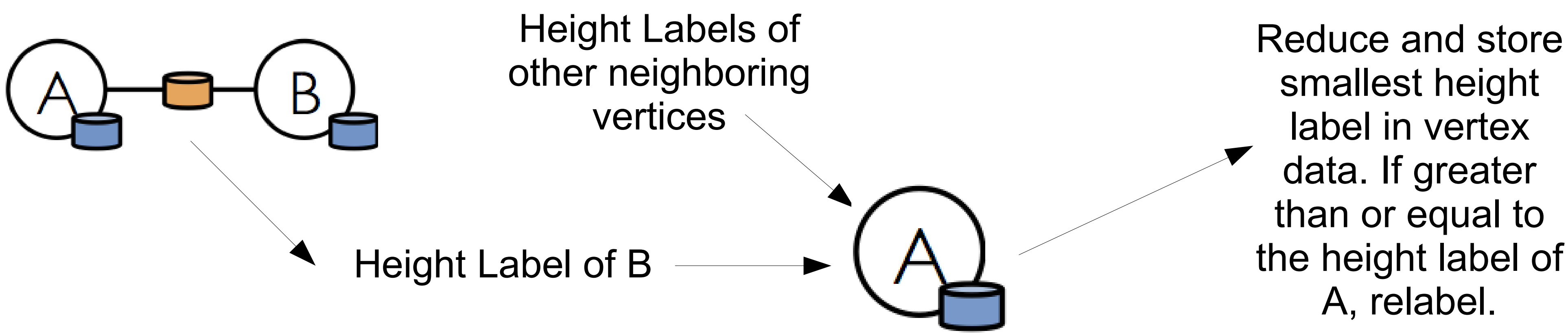
- While mapping, find the lowest neighboring height label.

Relabeling Visualization

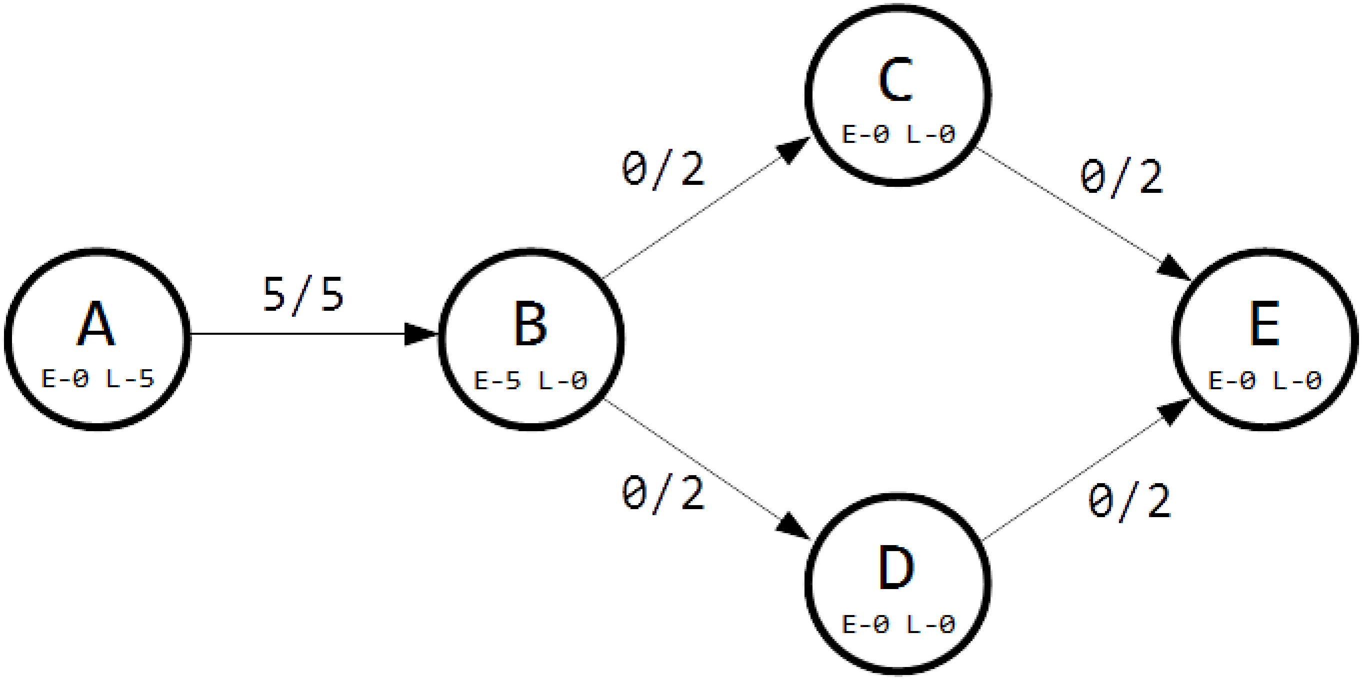
“Surveying” Step



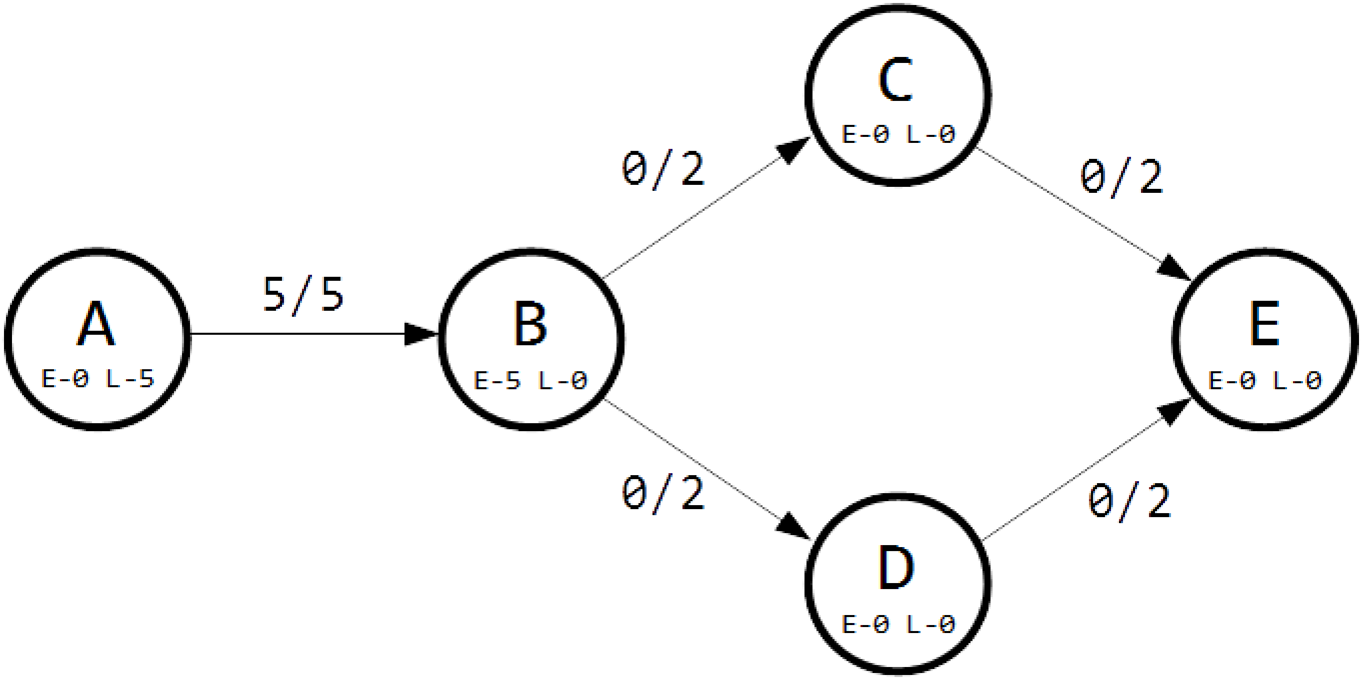
“Execution” Step

No actions, relabeling is already complete.

Simple Example



Simple Example: Iteration 1 - “Surveying”




```
From AB to B – (Map(), 5)
From BC to B – (Map(), 0)
From BD to B – (Map(), 0)
```

Reduce →

```
B – (Map(), 0)
```

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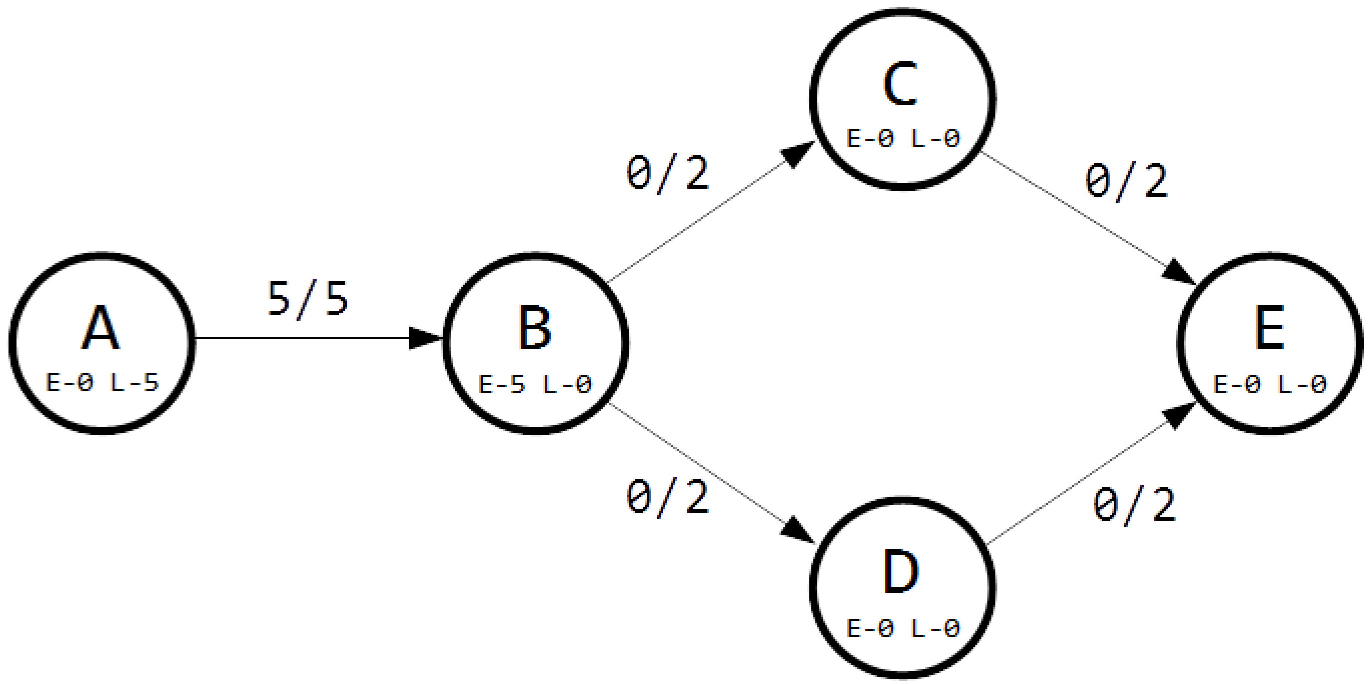


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From AB to B – (Map(), 5)
From BC to B – (Map(), 0)
From BD to B – (Map(), 0)
```

Reduce →

```
B – (Map(), 0)
```

```
B – (Map(), 0)
```

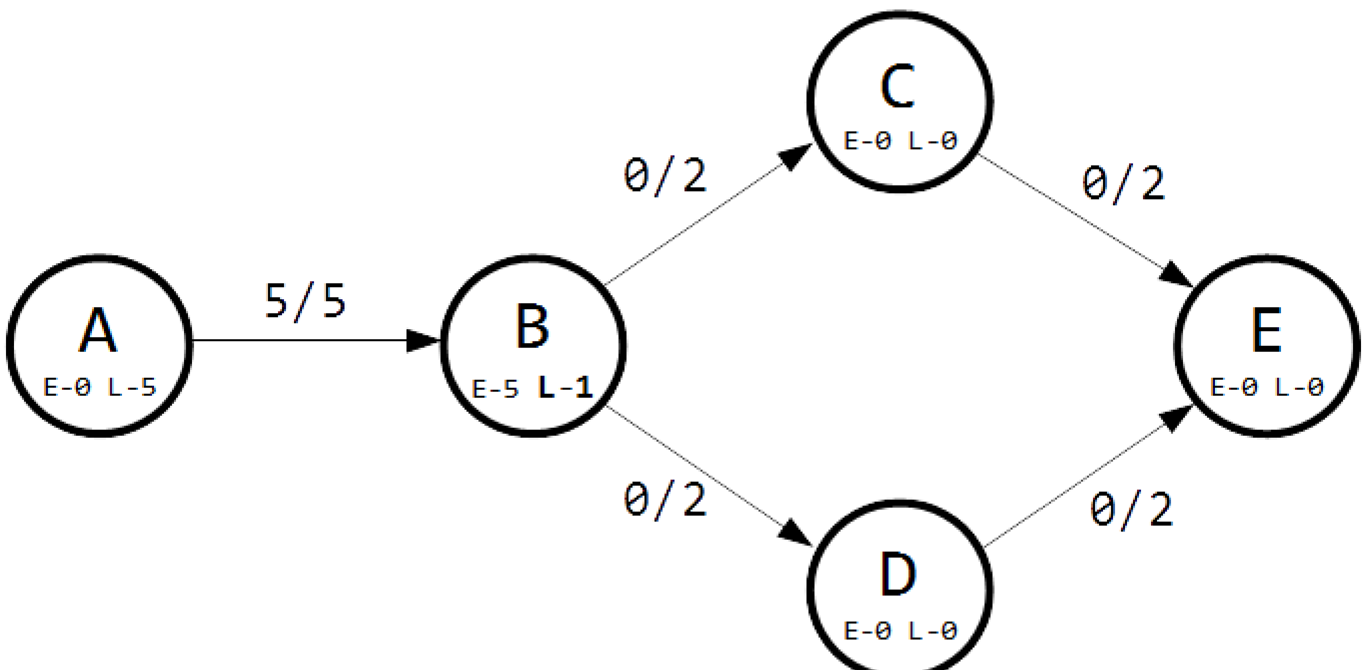
Vertex Program →

Relabel B to "1"

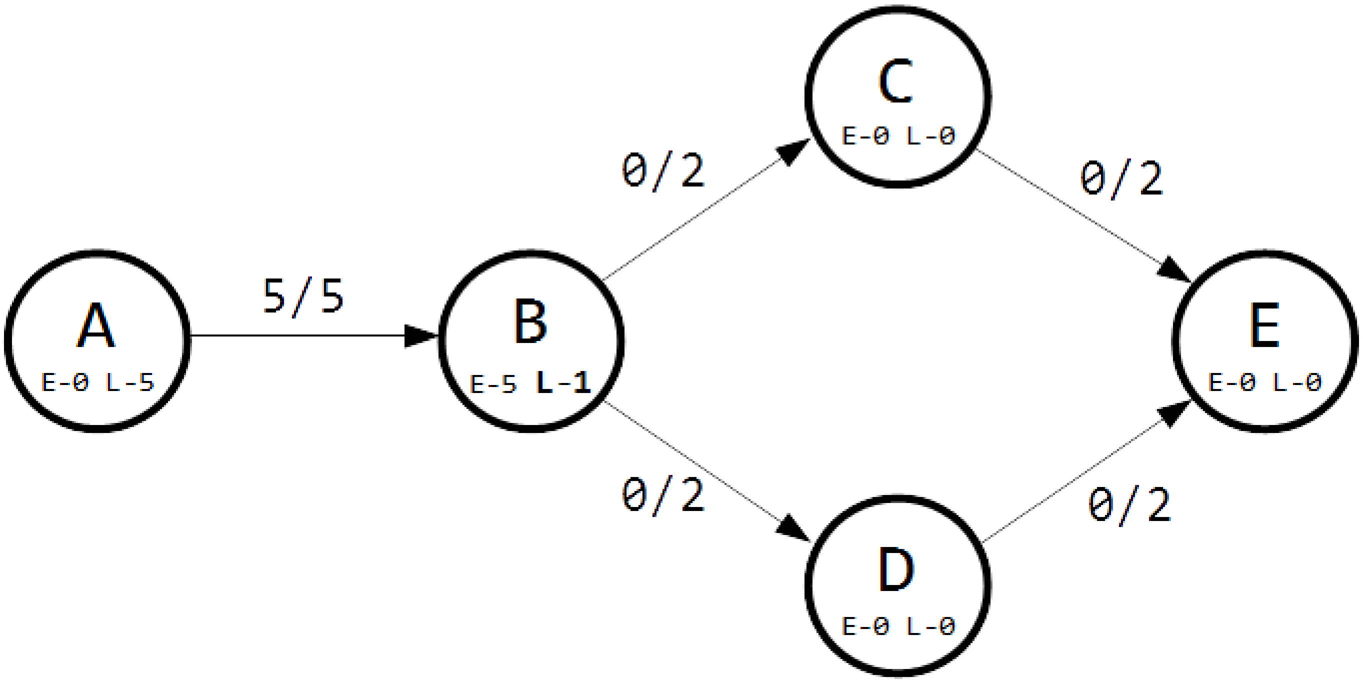
Simple Example: Iteration 1- “Execution”

No Possible Pushes, all vertices and edges
are mapped to their original values.

Resulting Graph:

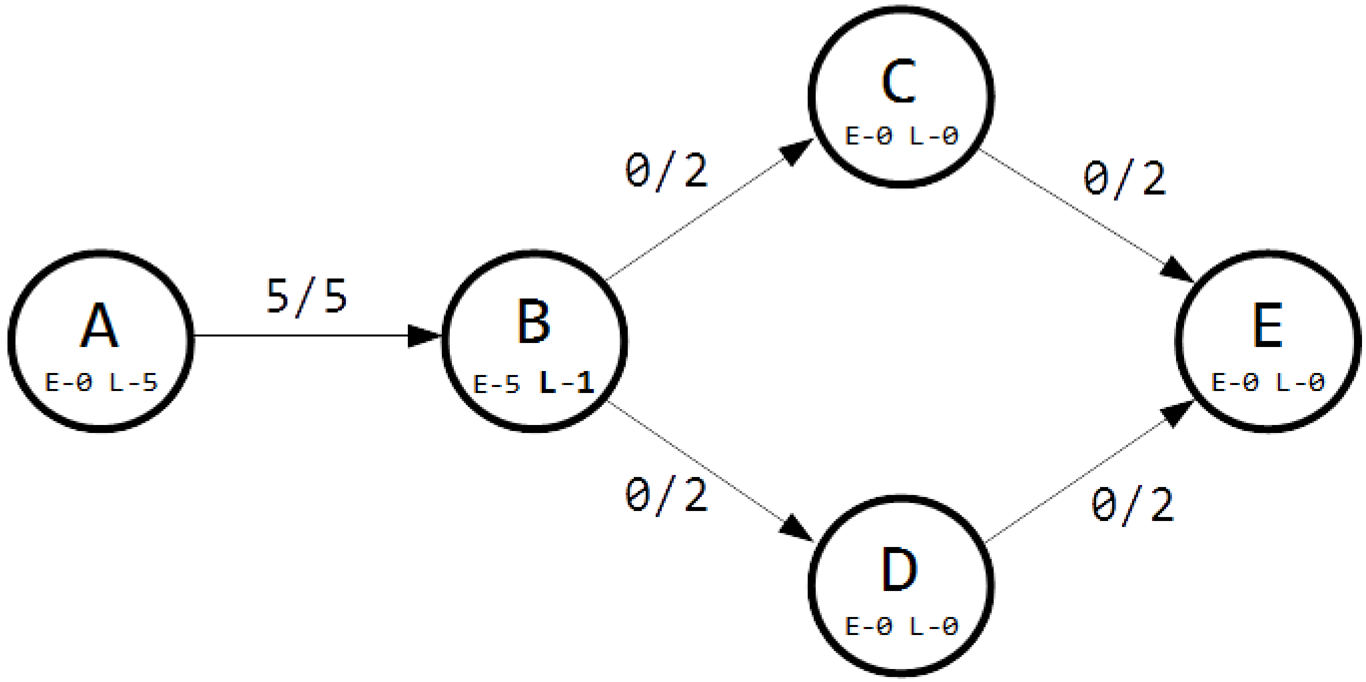


Simple Example: Iteration 2 - “Surveying”




```
From AB to B – (Map(), 5)
From BC to B – (Map((3L, true) → 2), 0)
From BD to B – (Map((4L, true) → 2), 0)
```

Simple Example: Iteration 2 - “Surveying”



From AB to B – (Map(), 5)
From BC to B – (Map((3L, true) → 2), 0)
From BD to B – (Map((4L, true) → 2), 0)

↓

B – (Map((3L, true) -> 2, (4L, true) -> 2), 0)

Simple Example: Iteration 2 - “Surveying” Vertex Program

$$\texttt{B – (Map(}\underbrace{\texttt{(3L, true) -> 2}}_{\text{Push 1}}\texttt{, }\underbrace{\texttt{(4L, true) -> 2}}_{\text{Push 2}}\texttt{), 0)}$$

<u>Loop over possible pushes</u>

Push 1 (Excess at B = 5): → 5 >= 2, select push and subtract excess.

Push 2 (Excess at B = 3): → 3 >= 2, select push and subtract excess.

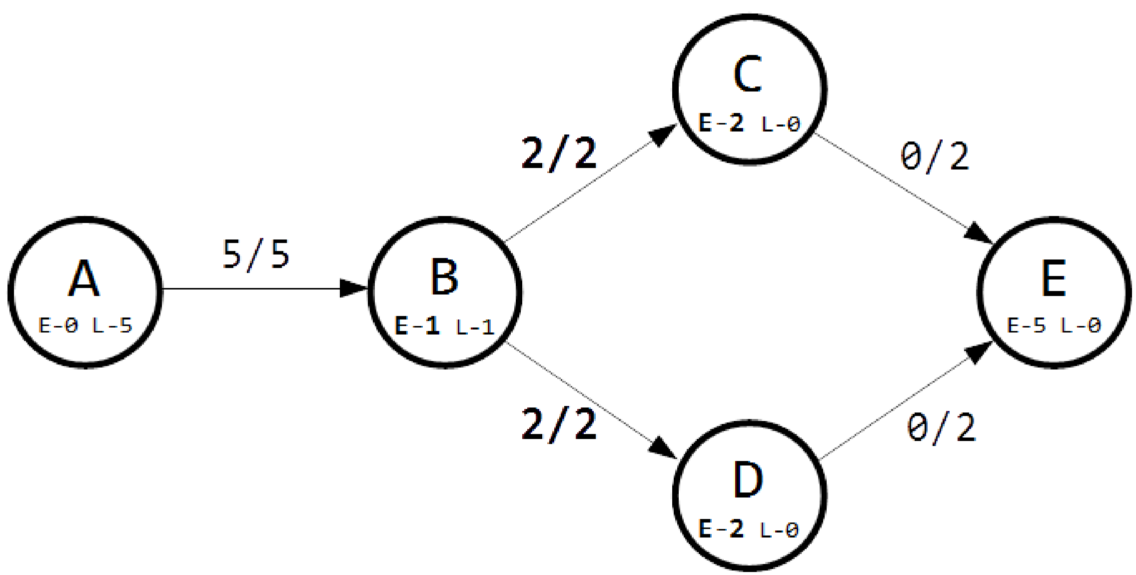
Data stored at Vertex B: **(1, 1, Map((3L, true) -> 2, (4L, true) -> 2))**

Simple Example: Iteration 2 - “Execution”

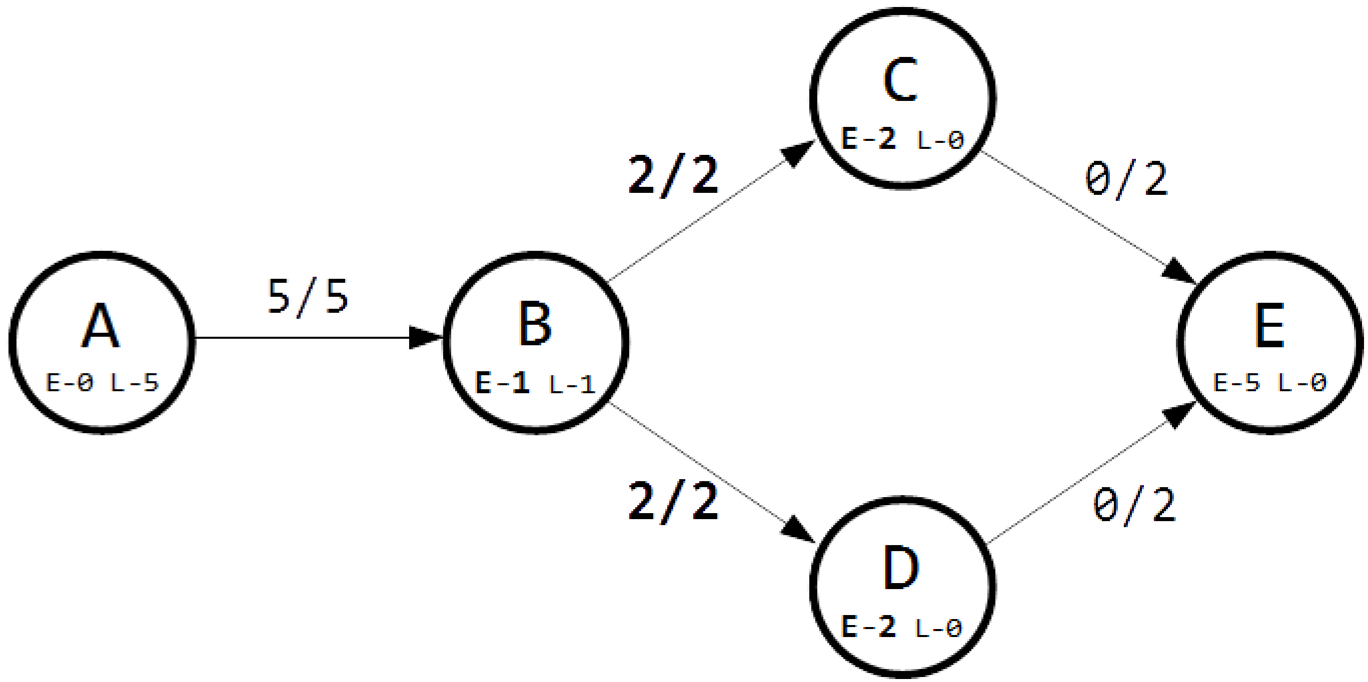
Vertex Data at B: **(1, 1, Map((3L, true) -> 2, (4L, true) -> 2))**

Update Edges → BC increases its flow by 2
BD increases its flow by 2

Update Vertices → C updates its excess by 2
D updates its excess by 2



Simple Example: Iteration 3 - “Surveying”




```
From AB to B – (Map(), 5)
From BC to C – (Map(), 1)
From BD to D – (Map(), 1)
From CE to C – (Map(), 0)
From DE to D – (Map(), 0)
```

Reduce →

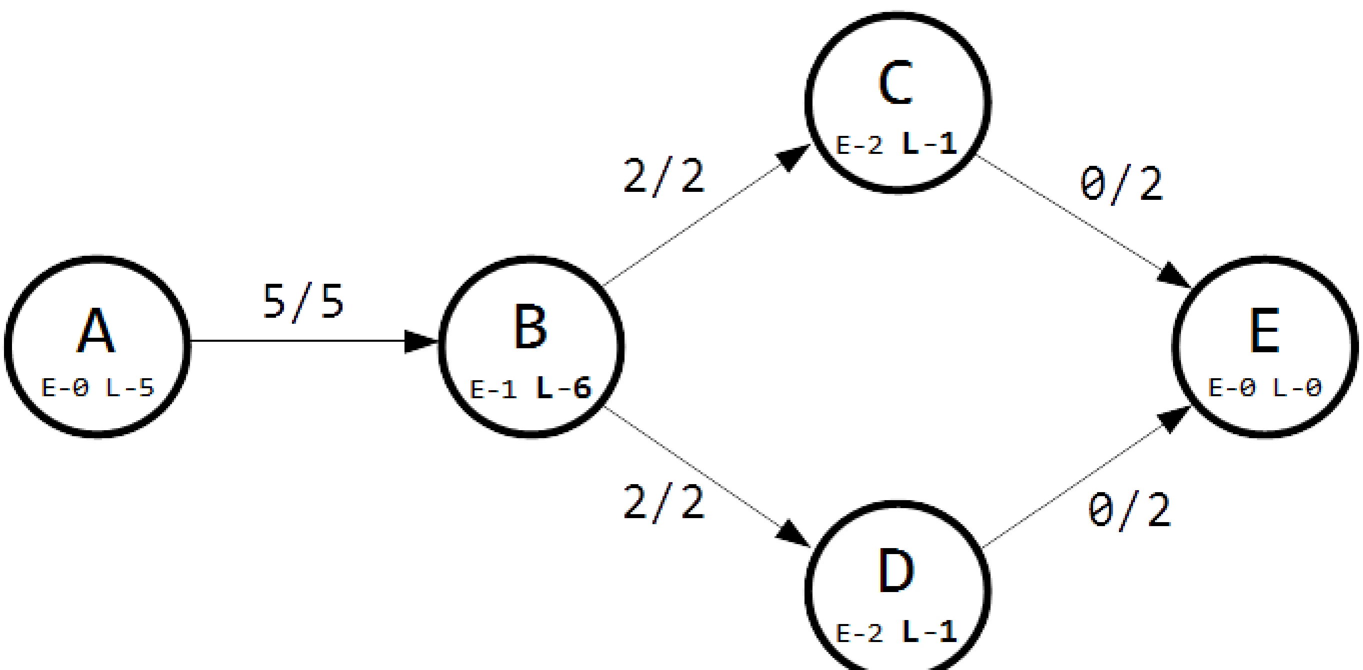
```
B – (Map(), 5)
C – (Map(), 0)
D – (Map(), 0)
```

→ All three vertices relabel

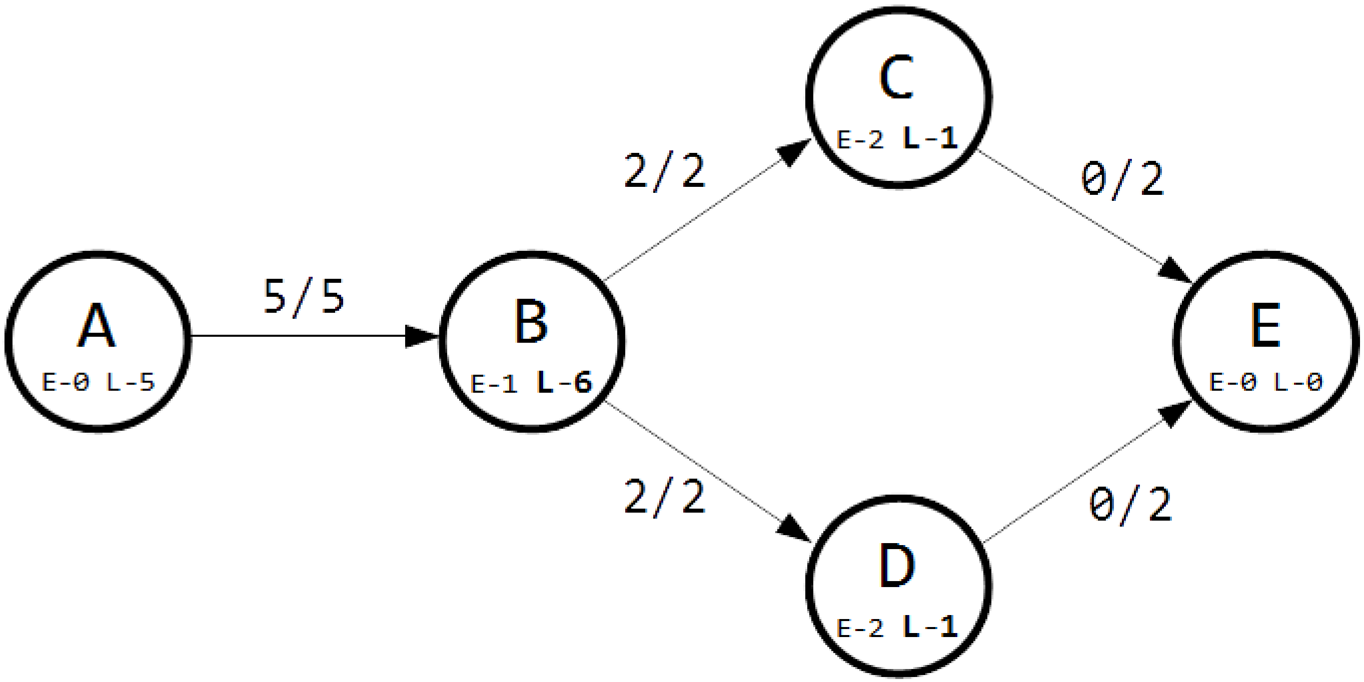
Simple Example: Iteration 3 - "Execution"

No Possible Pushes, all vertices and edges are mapped to their original values.

Resulting Graph:



Simple Example: Iteration 4 - “Surveying”




```
From AB to B – (Map((1L, false) → 5), 5)
From BC to C – (Map(), 1)
From BD to D – (Map(), 1)
From CE to C – (Map((5L, true) → 2), 0)
From DE to D – (Map((5L, true) → 2), 0)
```

Reduce →

```
B – (Map((1L, false) → 5), 5)
C – (Map((5L, true) → 2), 0)
D – (Map((5L, true) → 2), 0)
```

Simple Example: Iteration 4 - “Surveying” Vertex Program

Messages		Stored Vertex Data
`B – (Map((1L, false) → 5), 5)`	$\xrightarrow{\text{Excess of 1}}$	`(0, 6, Map((1L, false) → 1))`

Simple Example: Iteration 4 - “Surveying” Vertex Program

Messages		Stored Vertex Data
`B – (Map((1L, false) → 5), 5)`	Excess of 1 →	`(0, 6, Map((1L, false) → 1))`
`C – (Map((5L, true) → 2), 0)`	Excess of 2 →	`(0, 1, Map((5L, true) → 2))`

Simple Example: Iteration 4 - “Surveying” Vertex Program

Messages		Stored Vertex Data
`B – (Map((1L, false) → 5), 5)`	Excess of 1 →	`(0, 6, Map((1L, false) → 1))`
`C – (Map((5L, true) → 2), 0)`	Excess of 2 →	`(0, 1, Map((5L, true) → 2))`
`D – (Map((5L, true) → 2), 0)`	Excess of 2 →	`(0, 1, Map((5L, true) → 2))`

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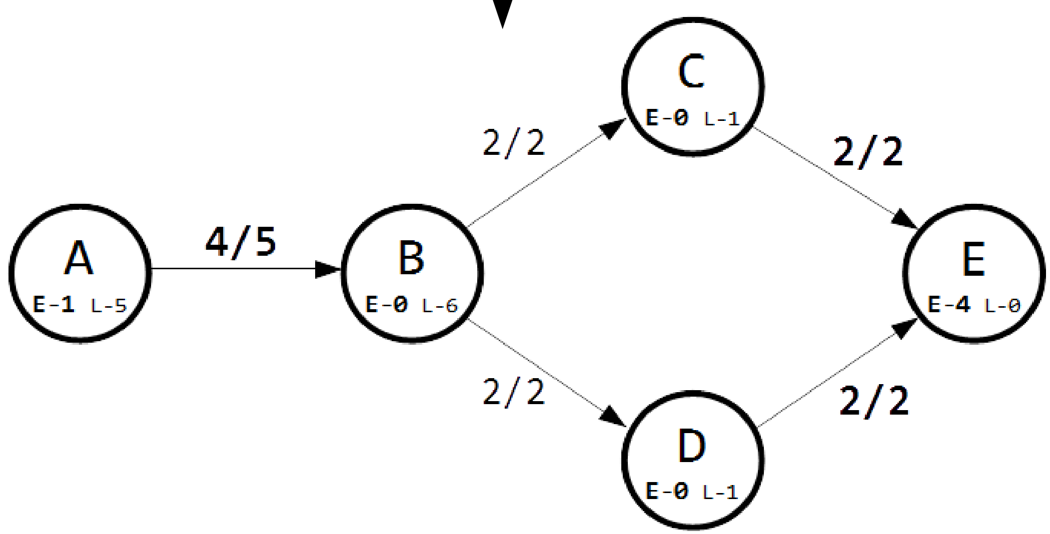
`C – (Map((5L, true) → 2), 0)`	Excess of 2 →	`(0, 1, Map((5L, true) → 2))`
`D – (Map((5L, true) → 2), 0)`	Excess of 2 →	`(0, 1, Map((5L, true) → 2))`

Pushed flow is subtracted from the excess at vertices B, C, and D

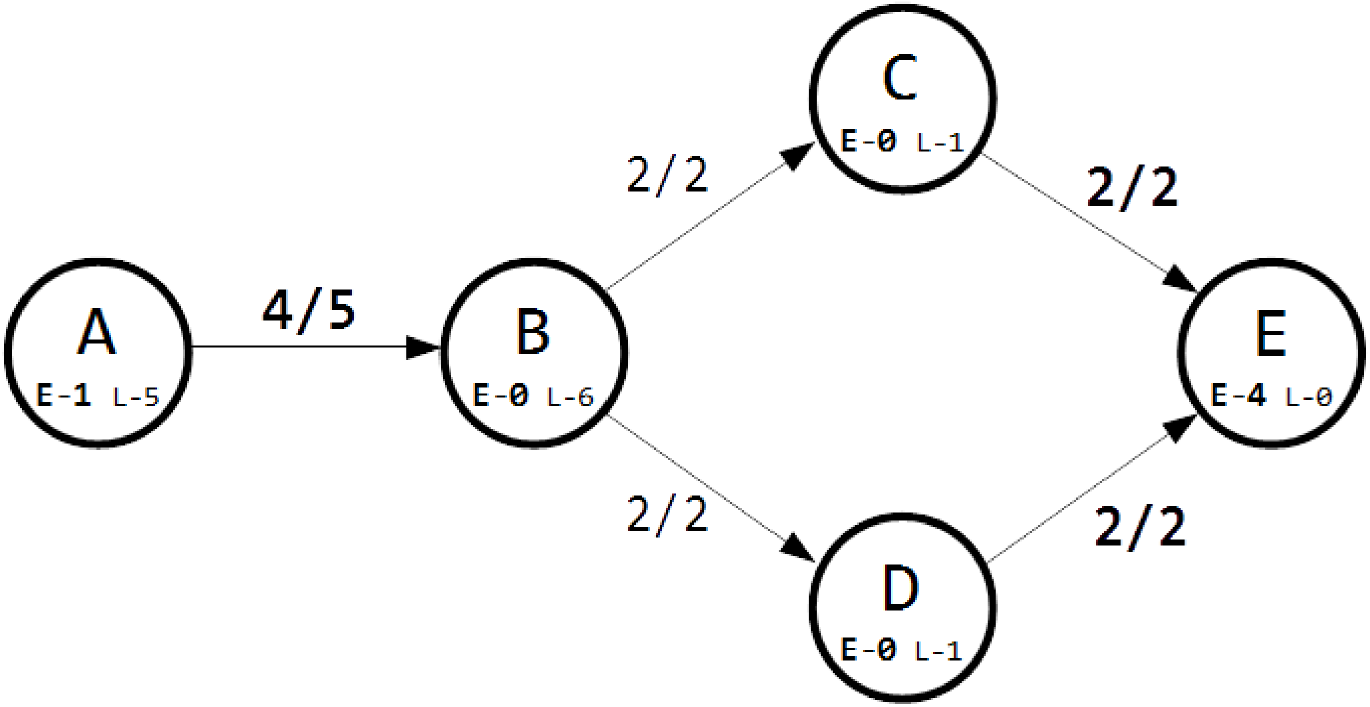
Simple Example: Iteration 4 - “Execution”

```
B - (0, 6, Map((1L, false) → 1))
C - (0, 1, Map((5L, true) → 2))
D - (0, 1, Map((5L, true) → 2))
```

Edges (CE, DE, AB) and
Vertices (A, E) Update

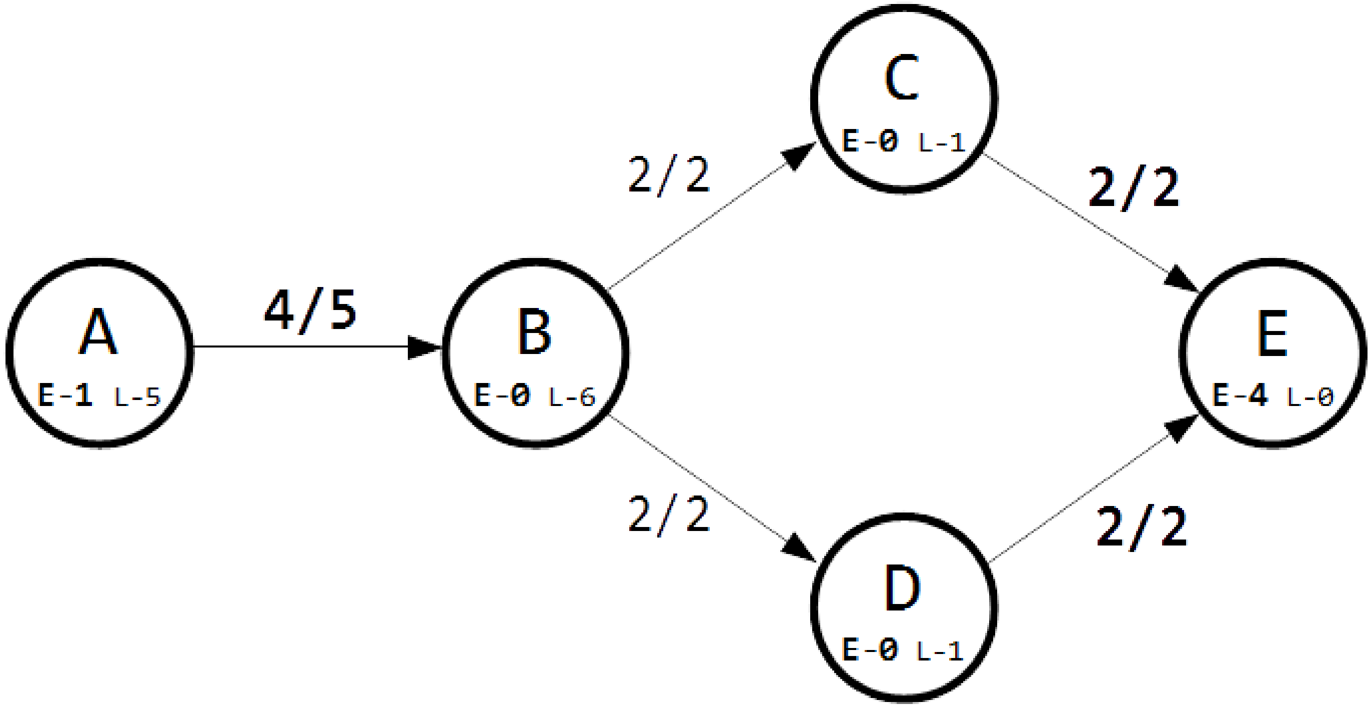


Simple Example: Iteration 5



- No excess in the graph (excluding the source and sink) leads to no messages.

Simple Example: Iteration 5



- No excess in the graph (excluding the source and sink) leads to no messages.
- Main execution loop terminates, and the maximum flow has been found.

Checkpointing

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- RDD lineage grows with each iteration, eventually causing a stack overflow.
- A checkpoint saves the RDD to an HDFS file and truncates the lineage entirely.
- Implemented by simply calling the checkpoint method after a set number of iterations.

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- Simply call the cache method on the graph.

Restricted Active Set

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- Was able to specify an RDD that specified which Triplets should be included.
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  - Could provide benefit with costly methods

Experimentation

- Used Amazon Elastic MapReduce services to run on a cluster (2 c3.xlarge instances).

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- Aimed to verify correctness and observe the effects of the variations mentioned.

Datasets (.bk files)

- Single-line → Contrived graph of 500 chained vertices
  - 499 edges
- Parallel-5-5 → Contrived graph branching at factor of 5
  - 3900 edges
- Parallel-12-5 → Contrived graph branching at factor of 12
  - 271440 edges
- RMF-wide → Smallest of benchmarks obtained online.
  - 93178 edges

Caching vs. Non-caching Results

	single-line (s)	RMF-wide 200 iter. (s)	parallel-5-5 (s)	parallel-12-5 (s)
Base	750.736	413.838	16.207	118.728
Cache	522.527	306.654	13.432	92.042

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- Both contrived parallel datasets complete in the same number of iterations.

Checkpointing Intervals Results

	single-line (s)	RMF-wide 200 iter. (s)	parallel-5-5 (s)	parallel-12-5 (s)
10 iterations	456.626	287.349	15.505	94.515
25 iterations	427.684	314.942	13.251	92.612
50 iterations	522.527	341.530	13.432	92.042

- Expectation was that more frequent checkpointing would always hurt performance.

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- Results seem to be unpredictable, and the middle (25 iterations) option appears to be best overall.

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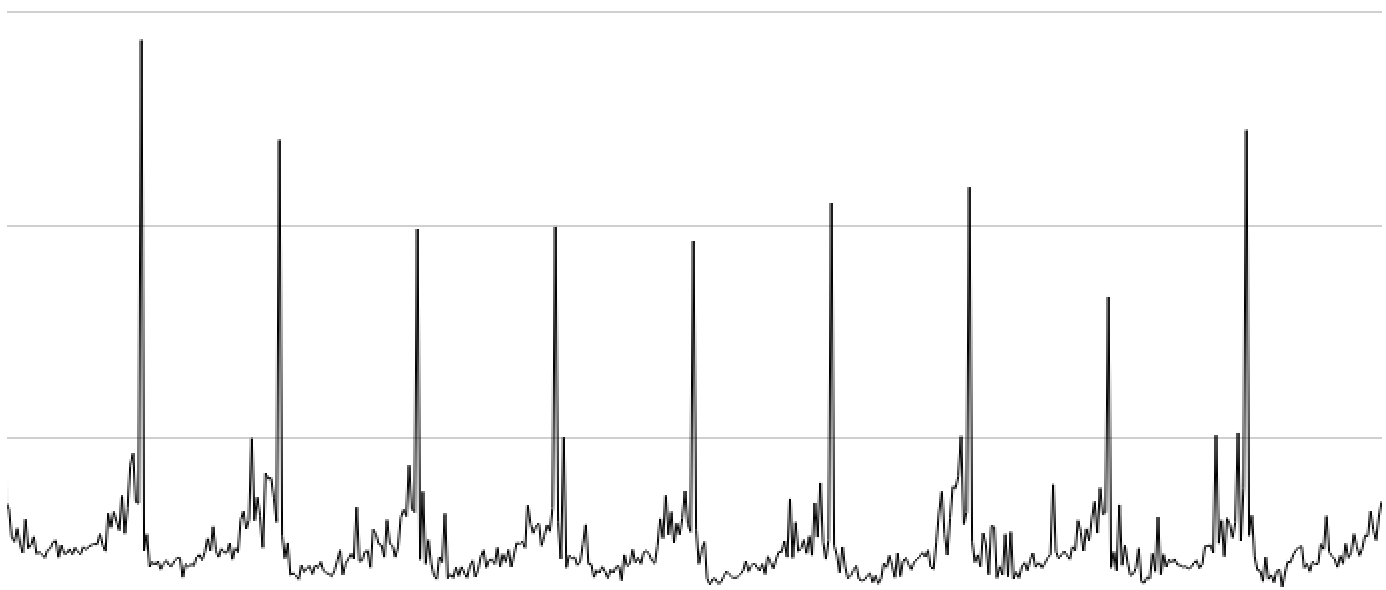

Iteration duration over time (spikes show checkpointing)

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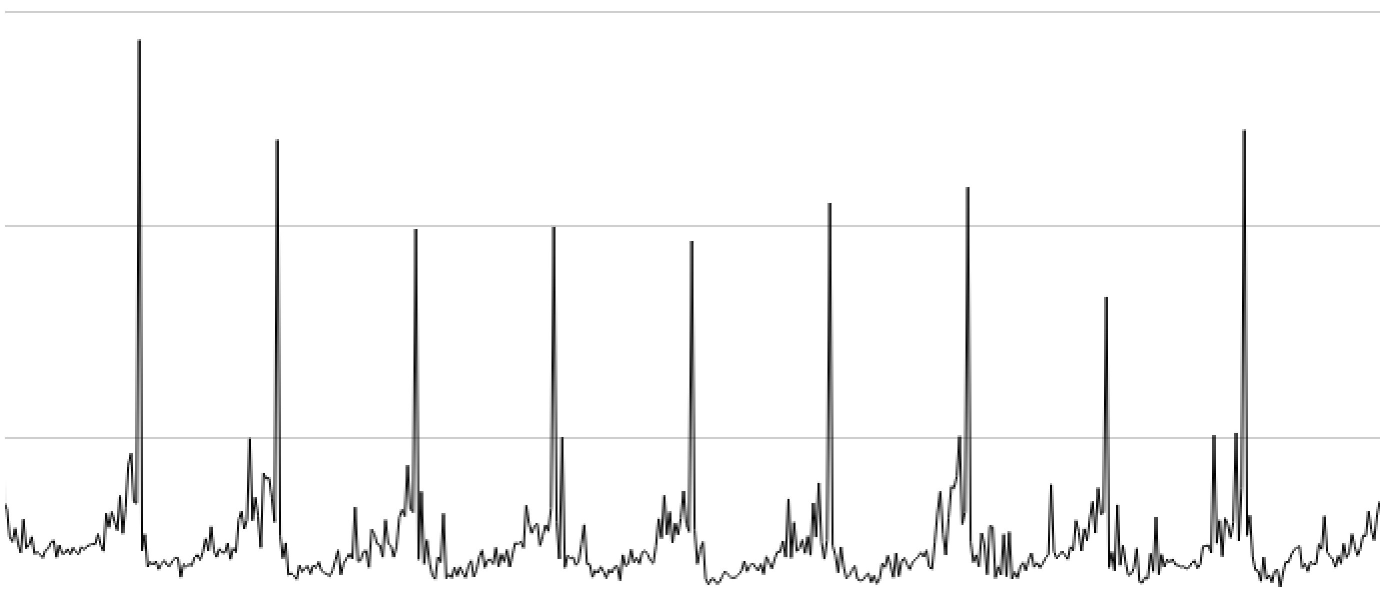

Iteration duration over time (spikes show checkpointing)

- Results show that iterations tend to grow longer in between checkpointing intervals.

- May indicate that there is some balance between the cost of checkpointing and the cost of increased lineage.

Scaling and Amazon Inconsistencies

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> "The performance of Amazon machine instances is sometimes fast, sometimes slow, and sometimes absolutely abysmal."
>
> - Blog article "Benchmarking Amazon EC2: The wacky world of cloud performance"

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  - Possibility of condensing MapReduce steps

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  - What is the impact that scaling the cluster has on performance?

- Algorithm optimization
  - Possibility of condensing MapReduce steps
  - Compare data structure selections

Possible Future Work

- Scaling and verification of approach
  - How does the solution perform in a truly “big data” context?
  - What is the impact that scaling the cluster has on performance?
- Algorithm optimization
  - Possibility of condensing MapReduce steps
  - Compare data structure selections
  - Explore manual uncaching

Open Questions

GraphX Code - Initialization

```scala
// Define types
type VertexPushMap = Map[(VertexId, Boolean), Int]
type EdgeData = (Int, Int)
type VertexData = (Int, Int, VertexPushMap)
type SurveyMessage = (VertexPushMap, Int)

// Initialize the graph
var activeMessages = 1
var iteration = 1

// Build graph
val vertexArray = vertexBuffer.toArray
val edgeArray = edgeBuffer.toArray
val vertexRDD: RDD[(VertexId, VertexData)] = sc.parallelize(vertexArray)
val edgeRDD: RDD[Edge[EdgeData]] = sc.parallelize(edgeArray)
var graph = Graph(vertexRDD, edgeRDD)
```

GraphX Code - “Surveying” MapReduce

```
// "Surveying" MapReduce step
val eligiblePushesRDD = graph.aggregateMessages[SurveyMessage] (
  // Map: Send message if vertex has excess
  edgeContext => {

    // Make sure not to push from sink or source
    if (edgeContext.srcId != sinkId && edgeContext.srcId != sourceId) {
      // If a residual edge exists from source to destination
      if (edgeContext.attr._2 > edgeContext.attr._1) {
        // If source has an excess
        if (edgeContext.srcAttr._1 > 0) {
          // If source has height one greater than destination
          if (edgeContext.srcAttr._2 == (edgeContext.dstAttr._2 + 1)) {
            // Push is possible, send message to source containing push information
            val pushAmount = math.min(edgeContext.attr._2 - edgeContext.attr._1, edgeContext.srcAttr._1)
            edgeContext.sendToSrc((Map((edgeContext.dstId, true) -> pushAmount), edgeContext.dstAttr._2))
          } else {
            edgeContext.sendToSrc((Map(), edgeContext.dstAttr._2))
          }
        }
      }
    }

    (Repeated in other direction along the edge)

  },
  // Reduce: Concatenate into map of all possible pushes, keep track of relabel eligibility
  (a, b) => {
    (a._1 ++ b._1, math.min(a._2, b._2))
  }
)
```

GraphX Code - “Surveying” MapReduce

```
// "Surveying" MapReduce step
val eligiblePushesRDD = graph.aggregateMessages[SurveyMessage] (
```

Map

```
  // Map: Send message if vertex has excess
  edgeContext => {

    // Make sure not to push from sink or source
    if (edgeContext.srcId != sinkId && edgeContext.srcId != sourceId) {
      // If a residual edge exists from source to destination
      if (edgeContext.attr._2 > edgeContext.attr._1) {
        // If source has an excess
        if (edgeContext.srcAttr._1 > 0) {
          // If source has height one greater than destination
          if (edgeContext.srcAttr._2 == (edgeContext.dstAttr._2 + 1)) {
            // Push is possible, send message to source containing push information
            val pushAmount = math.min(edgeContext.attr._2 - edgeContext.attr._1, edgeContext.srcAttr._1)
            edgeContext.sendToSrc((Map((edgeContext.dstId, true) -> pushAmount), edgeContext.dstAttr._2))
          } else {
            edgeContext.sendToSrc((Map(), edgeContext.dstAttr._2))
          }
        }
      }
    }

    (Repeated in other direction along the edge)

  },
```

Reduce

```
  // Reduce: Concatenate into map of all possible pushes, keep track of relabel eligibility
  (a, b) => {
    (a._1 ++ b._1, math.min(a._2, b._2))
  }
)
```

GraphX Code - “Surveying” Vertex Program

```
graph = graph.outerJoinVertices(eligiblePushesRDD) {
  (id: VertexId, data: VertexData, msg: Option[SurveyMessage]) => {
    // Store empty map if no messages
    if (msg.isEmpty) {
      (data._1, data._2, Map[(VertexId, Boolean), Int]())
    } else if (msg.get._2 >= data._2) {
      // Eligible for relabel
      (data._1, msg.get._2 + 1, Map[(VertexId, Boolean), Int]())
    } else {
      // Add pushes until no excess remains or pushes are exhausted
      var excess = data._1
      val selectedPushes = scala.collection.mutable.Map[(VertexId, Boolean), Int]()

      // Select pushes until flow is gone, break once no flow is remaining.
      breakable {
        msg.get._1.foreach(pushData => {
          val dstId = pushData._1._1
          val forwardPush = pushData._1._2
          val pushAmount = pushData._2
          if (excess > 0) {
            val selectedPushAmount = math.min(pushAmount, excess)
            excess -= selectedPushAmount
            selectedPushes((dstId, forwardPush)) = selectedPushAmount
          } else {
            break
          }
        })
      }

      (excess, data._2, selectedPushes.toMap)
    }
  }
}
```

GraphX Code - “Surveying” Vertex Program

No messages

No pushes, relabel

Possible pushes, select based on excess

```
graph = graph.outerJoinVertices(eligiblePushesRDD) {
  (id: VertexId, data: VertexData, msg: Option[SurveyMessage]) => {
    // Store empty map if no messages
    if (msg.isEmpty) {
      (data._1, data._2, Map[(VertexId, Boolean), Int]())
    } else if (msg.get._2 >= data._2) {
      // Eligible for relabel
      (data._1, msg.get._2 + 1, Map[(VertexId, Boolean), Int]())
    } else {
      // Add pushes until no excess remains or pushes are exhausted
      var excess = data._1
      val selectedPushes = scala.collection.mutable.Map[(VertexId, Boolean), Int]()

      // Select pushes until flow is gone, break once no flow is remaining.
      breakable {
        msg.get._1.foreach(pushData => {
          val dstId = pushData._1._1
          val forwardPush = pushData._1._2
          val pushAmount = pushData._2
          if (excess > 0) {
            val selectedPushAmount = math.min(pushAmount, excess)
            excess -= selectedPushAmount
            selectedPushes((dstId, forwardPush)) = selectedPushAmount
          } else {
            break
          }
        })
      }

      (excess, data._2, selectedPushes.toMap)
    }
  }
}
```

GraphX Code - “Execution” MapReduce

```
val executedPushesRDD = graph.aggregateMessages[Int] (
  // Map: Send push information to vertices that received flow
  edgeContext => {

    // Check if destination vertex id is in the source's push map
    if (edgeContext.srcAttr._3.contains((edgeContext.dstId, true))) {
      val pushAmount: Int = edgeContext.srcAttr._3((edgeContext.dstId, true))
      edgeContext.sendToDst(pushAmount)
    }

    // Check if source vertex id is in the destinations's push map
    if (edgeContext.dstAttr._3.contains((edgeContext.srcId, false))) {
      val pushAmount: Int = edgeContext.dstAttr._3((edgeContext.srcId, false))
      edgeContext.sendToSrc(pushAmount)
    }

  },
  // Reduce: Combine all incoming flow into a single total
  (a, b) => {
    a + b
  }
)
```

GraphX Code - “Execution” MapReduce

Map

Reduce

```
val executedPushesRDD = graph.aggregateMessages[Int] (
  // Map: Send push information to vertices that received flow
  edgeContext => {

    // Check if destination vertex id is in the source's push map
    if (edgeContext.srcAttr._3.contains((edgeContext.dstId, true))) {
      val pushAmount: Int = edgeContext.srcAttr._3((edgeContext.dstId, true))
      edgeContext.sendToDst(pushAmount)
    }

    // Check if source vertex id is in the destinations's push map
    if (edgeContext.dstAttr._3.contains((edgeContext.srcId, false))) {
      val pushAmount: Int = edgeContext.dstAttr._3((edgeContext.srcId, false))
      edgeContext.sendToSrc(pushAmount)
    }

  },
  // Reduce: Combine all incoming flow into a single total
  (a, b) => {
    a + b
  }
)
```

GraphX Code - “Execution” Vertex Program

```
// Update excess values
graph = graph.outerJoinVertices(executedPushesRDD) {
  (id: VertexId, data: VertexData, msg: Option[Int]) => {
    // Add pushed flow to vertex
    (data._1 + msg.getOrElse(0), data._2, data._3)
  }
}
```