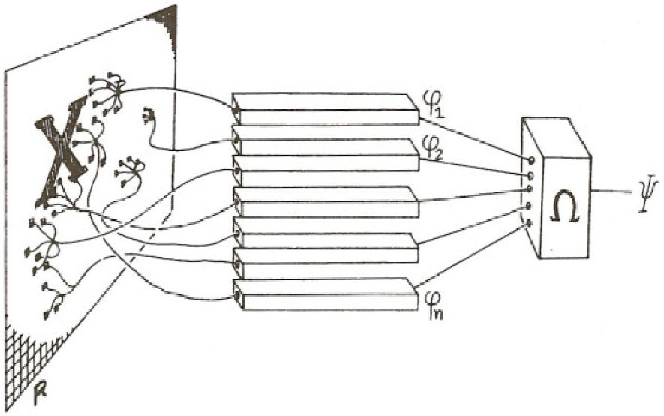
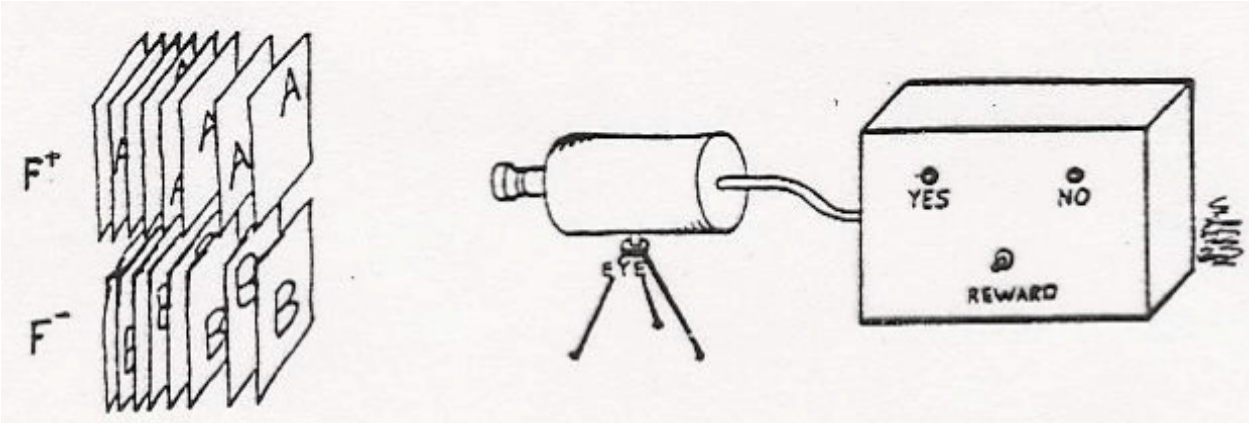
# Mike's Brief History Of Machine Learning

## 1962

**Frank Rosenblatt, *Principles of Neurodynamics: Perceptrons and the Theory of Brain Mechanisms***

**Perceptron can learn anything you can program it to do.**



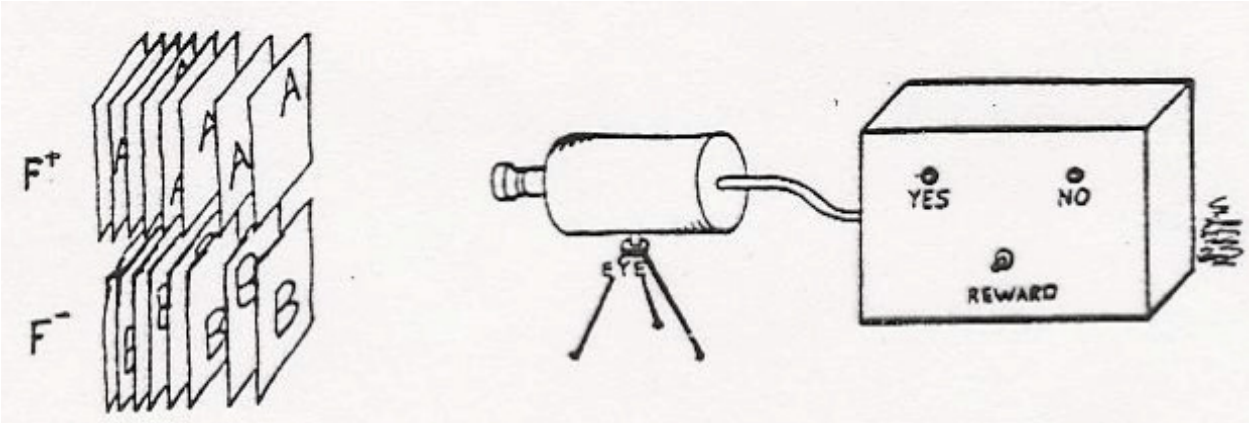



# Mike's Brief History Of Machine Learning

## 1969

Minsky & Papert, *Perceptrons*

There are many things a perceptron can't in principle learn to do



# Mike's Brief History Of Machine Learning

## 1970-1985

Attempts to develop symbolic rule discovery algorithms

## 1986

Rumelhart, Hinton, & Williams, *Back propagation*

Overcame many of the Minsky & Papert objections

## 1990-2000

Statisticians

# Bayesian Optimization: From A/B Testing To A-Z Testing

Robert V. Lindsey, Brett Roads,
Mohammad Khajah, Michael Mozer
Department of Computer Science
University of Colorado, Boulder

Harold Pashler
Department of Psychology
UC San Diego

# A/B Testing

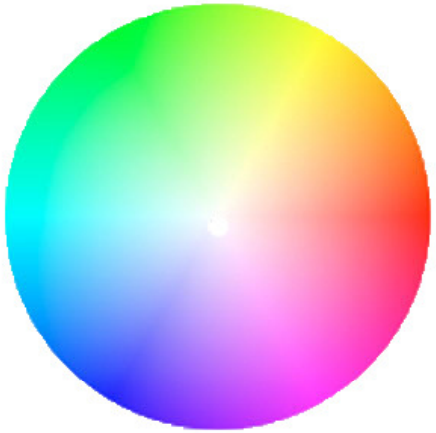
Randomly assign web-page visitors to one of two conditions, A or B

Serve A or B version of web page according to condition

Measure which condition leads to better results

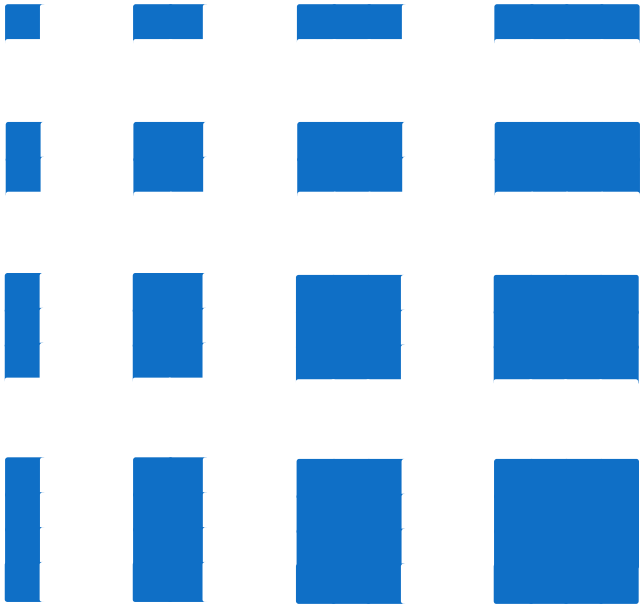
# A/B Testing On Steroids

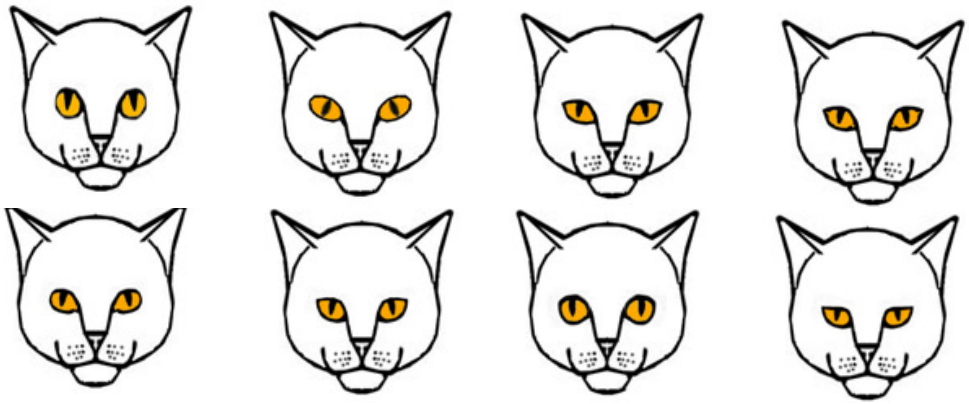
**Suppose we could compare not just two or a small number of options…**



**But a continuum of options…**

**As efficiently as we compared 2.**





# From Your World To Mine

**A/B testing isn't used just in marketing and high tech companies.**

**A/B testing is the core technique used in science.**

- **known as a *randomized controlled experiment.***

# Randomized Controlled Experiments In Psychology

E.g., distributed-practice effect

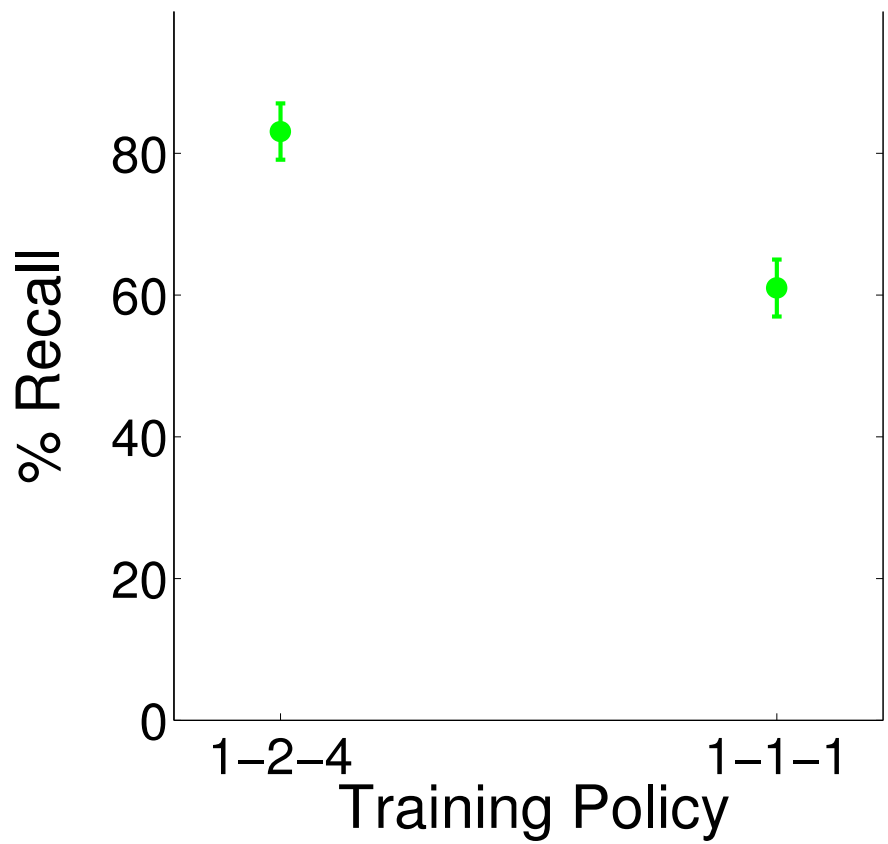
- massed vs. spaced practice

Propose several spaced conditions to compare

- Equal: 1 – 1 – 1
- Increasing: 1 – 2 – 4

Run many subjects in each condition

Perform statistical analyses to establish reliable difference between conditions

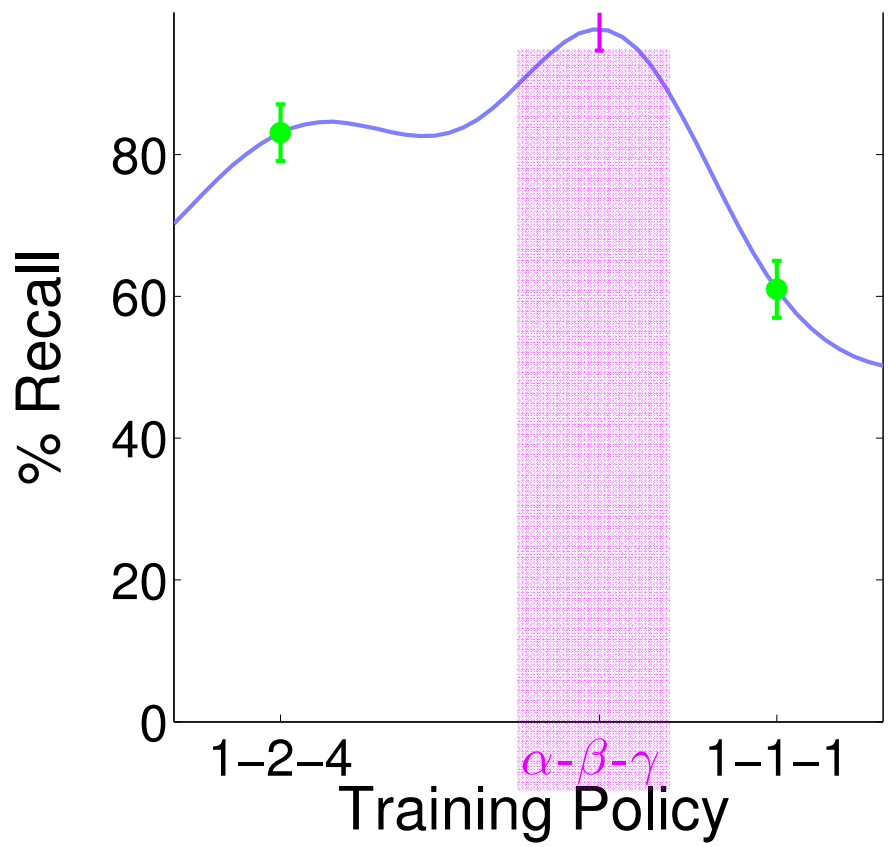


# What Researchers Really Want To Do

**Find the best study schedule *(training policy)***

**Abscissa: space of all training policies**

**Performance function defined over policy space**



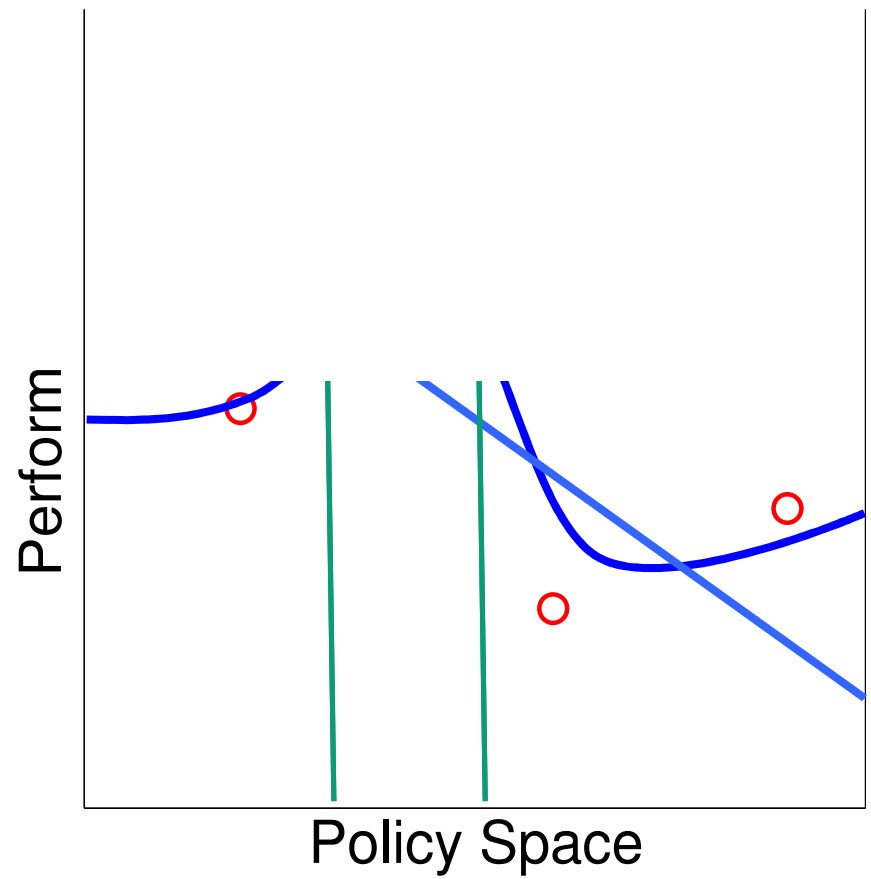
# Approach

**Perform single-subject experiments at selected points in policy space (o)**

**Use curve fitting (function approximation) techniques to estimate shape of the performance function**

**Given current estimate, select *promising* policies to evaluate next.**

- **promising = has potential to be the optimum policy**



# Gaussian Process Regression

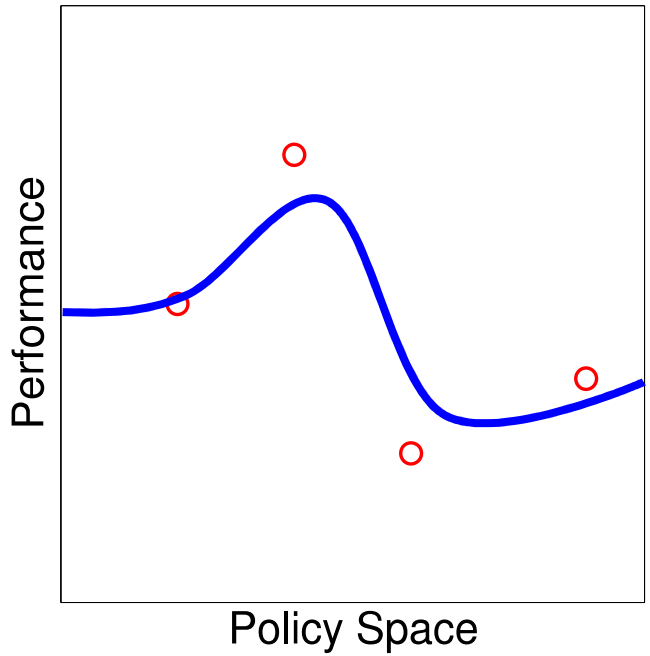
Assumes only that functions are smooth

- How smooth is determined by the data

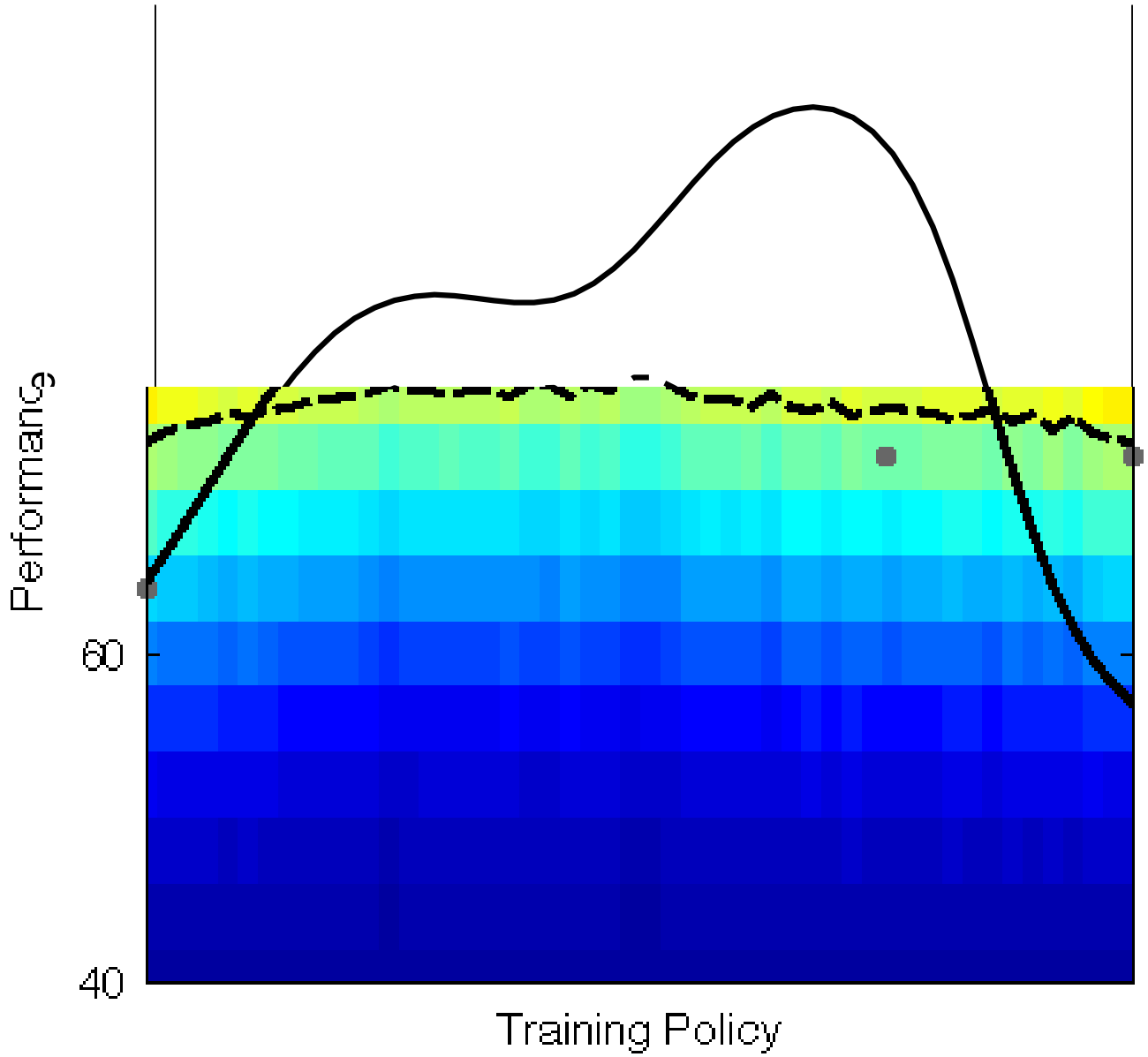
Uses data efficiently

Accommodates noisy data

Produces estimates of both function shape and uncertainty



# Simulated Experiment



# 1 Policy Selection Heuristic

I propose the following heuristic for choosing the next training policy to evaluate. Let the random variable $\pi_x$ be the population average performance at policy $x$,

$$\pi_x = .5 + .5\frac{1}{1+\exp(-f_x)} \tag{1}$$

We can calculate the expectation $\mu_x = \mathrm{E}\left[\pi_x\right]$ using samples from the posterior predictive distribution of the GP $\vec{f}$. I propose we choose the next training policy $\hat{x}$ based on

$$\hat{x} = \arg\max_x \mathrm{E}\left[(m\mu_x - \pi_x)^2\right] \tag{2}$$

where $0 \leq m \leq 1$. Pure exploitation ($m = 0$) and pure exploration ($m = 1$) are the extreme cases of this:

$$\hat{x} = \arg\max_x \mathrm{E}\left[\pi_x^2\right] = \arg\max_x \mathrm{E}\left[\pi_x\right] \quad \text{when m=0} \tag{3}$$

$$\hat{x} = \arg\max_x \mathrm{Var}\left[\pi_x\right] \quad \text{when m=1} \tag{4}$$

Thinking of this expectation as a weighted sum of squared-distances between $m\mu_x$ and $\pi$ (summed across possible $x$'s), $m$ lets us manipulate the magnitude of the distances while keeping the weights fixed. If $m$ is near 0, the distances are at their largest for large $\pi_x$ values. Hence, even if small $\pi_x$ are more highly weighted (ie more probable), the big squared-distance values of the less probable larger $\pi_x$'s will matter the most. Conversely, an $m$ near 1 places less emphasis on the squared-distances and more emphasis on their weights.

Assuming this is a sensible approach, I prefer it to a selection policy that uses the raw GP function estimates because of previously discussed issues associated with large uncertainty at extremely high or low GP values not mattering. Also, having a policy selection heuristic that's based loosely on our prediction of an observable variable seems better than using a prediction of an unobservable variable.

Also note that $\mathrm{Var}\left[\pi_x\right]$ goes to 0 as we run more and more experiments at policy $x$. We shouldn't get stuck choosing the same policy over and over again when $m > 0$.

# 2 Marginal Likelihood

## 2.1 Lemma

Given

$$\begin{aligned} p \mid \alpha, \beta &\sim \mathrm{Beta}(\alpha, \beta) && (5)\\ \tilde{p} &= .5 + .5p && (6)\\ n_c \mid p, n_\cdot &\sim \mathrm{Binomial}(\tilde{p}, n_\cdot) && (7) \end{aligned}$$

where, in our case, $n_\cdot$ is the number of test questions, $n_c$ is the number of correct responses made, $\tilde{p}$ is the subject's mean recall probability *corrected for chance guessing*. The marginal likelihood is

$$\mathrm{P}(n_c \mid \alpha, \beta) = 2^{-n}\binom{n}{n_c}\sum_{i=0}^{n_c}\binom{n_c}{i}\frac{\mathrm{B}(\alpha+i, n_\cdot+\beta-n_c)}{\mathrm{B}(\alpha,\beta)} \tag{8}$$

## 2.2 Proof

The chance-corrected likelihood equation is

$$\begin{aligned} L(n_c|n_\cdot, p) &= \binom{n_\cdot}{n_c}\tilde{p}^{\,n_c}(1-\tilde{p})^{n_\cdot-n_c} && (9)\\ &= \binom{n_\cdot}{k}.5^{n_\cdot}(1+p)^{n_c}(1-p)^{n_\cdot-n_c} && (10) \end{aligned}$$

The beta prior is

$$\pi(p|\alpha.\beta) = \frac{1}{\mathrm{B}(\alpha,\beta)}p^{\alpha-1}(1-p)^{\beta-1} \tag{11}$$

where B is the beta function. The marginal likelihood is defined as

$$\begin{aligned} P(n_c|\alpha,\beta,n_\cdot) &= \int_0^1 L(n_c|n_\cdot,p)\pi(p|\alpha,\beta)\,dp && (12)\\ &= 2^{-n_\cdot}\frac{1}{\mathrm{B}(\alpha,\beta)}\binom{n_\cdot}{n_c}\int_0^1(1+p)^{n_c}p^{\alpha-1}(1-p)^{\beta-1+n_\cdot-n_c}\,dp && (13) \end{aligned}$$

Because $n_c$ is an integer, we can apply the binomial theorem

$$\begin{aligned} P(n_c|\alpha,\beta,n_\cdot) &= 2^{-n_\cdot}\frac{1}{\mathrm{B}(\alpha,\beta)}\binom{n_\cdot}{n_c}\int_0^1\sum_{i=0}^{n_c}\binom{n_c}{i}p^i\,p^{\alpha-1}(1-p)^{\beta-1+n_\cdot-n_c}\,dp && (14)\\ &= 2^{-n_\cdot}\frac{1}{\mathrm{B}(\alpha,\beta)}\binom{n_\cdot}{n_c}\sum_{i=0}^{n_c}\binom{n_c}{i}\int_0^1 p^{\alpha+i-1}(1-p)^{\beta-1+n_\cdot-n_c}\,dp && (15) \end{aligned}$$

The integral in the summation is over an unnormalized $\mathrm{Beta}(\alpha+i, n+\beta-k)$ density. Therefore,

$$P(n_c|\alpha,\beta,n_\cdot) = 2^{-n_\cdot}\binom{n_\cdot}{n_c}\sum_{i=0}^{n_c}\binom{n_c}{i}\frac{\mathrm{B}(\alpha+i, n_\cdot+\beta-n_c)}{\mathrm{B}(\alpha,\beta)} \tag{16}$$

# 3 Inference

## 3.1 Model

The model we assume is

$$\begin{aligned} \mathbf{f} &\sim \mathrm{GP}(m(x), \Sigma(x,x')) && (17)\\ p_s \mid \alpha, f_s &\sim \mathrm{Beta}(\alpha, \alpha\exp(-f_s)) && (18)\\ \tilde{p}_s &= .5 + .5p_s && (19)\\ n_{cs} \mid p_s, n_{\cdot s} &\sim \mathrm{Binomial}(\tilde{p}_s, n_{\cdot s}) && (20) \end{aligned}$$

where $s$ is a subject index, $\alpha$ is a free parameter controlling inter-subject variability. Before performing inference, we analytically marginalize $p_s$ via Equation 8.

The model likelihood can be written as

$$\mathcal{L} = P(\mathbf{n}_c \mid \mathbf{f}) = \prod_s 2^{-n_{\cdot s}}\binom{n_{\cdot s}}{n_{cs}}\sum_{i=0}^{n_c}\binom{n_{cs}}{i}\frac{\mathrm{B}(\alpha+i, n_{ws}+\alpha e^{-f_s})}{\mathrm{B}(\alpha, \alpha e^{-f_s})} \tag{21}$$

where $n_{ws}$ is the number of wrong responses made by subject $s$. The prior follows a MVN density.

## 3.2 Gradient and Hessian

Let

$$z \equiv \alpha\left(\sum_{i=0}^{n_c}\binom{n_c}{i}\mathrm{B}(\alpha+i, n_\cdot+\beta+n_c)\right)^{-1} \tag{22}$$

We have

$$\frac{\partial}{\partial f_s}\log\mathcal{L} = ze^{-f_s}\Gamma(n_{ws}+\alpha e^{-f_s})\sum_{i=0}^{n_c}\binom{n_c}{i}\Gamma(\alpha+i)\frac{\Psi(n_{ws}+i+\alpha+\alpha e^{-f_s}) - \Psi(n_{ws}+\alpha e^{-f_s})}{\Gamma(n_{ws}+i+\alpha+\alpha e^{-f_s})} \tag{23}$$

where $\Psi$ is the digamma function.

$$\frac{\partial^2}{\partial f_s^2}\log\mathcal{L} = \ldots \tag{24}$$

## 3.3 Laplace Approximation

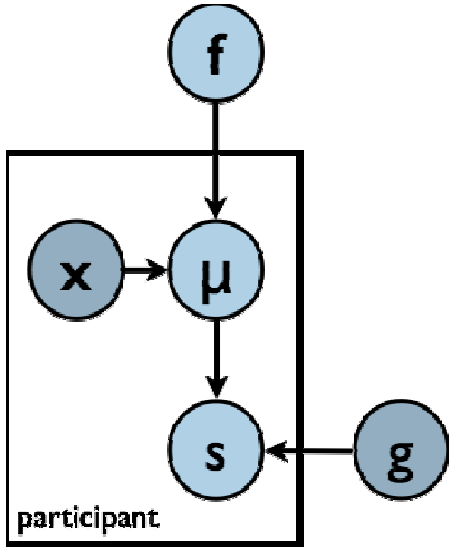
Unfinished section:

We can approximate the model's posterior distribution via a Gaussian centered at the mode

$$p(\mathbf{f}|\mathbf{n}_c) \sim q(\mathbf{f}|\mathbf{n}_c) = \mathcal{N}(\hat{\mathbf{f}}, (K^{-1}+W)^{-1}) \tag{25}$$

where $K$ is the covariance matrix of the data, $W \equiv -\nabla\nabla L$ is the (diagonal) Hessian, $\hat{\mathbf{f}}$ is the mode (maximum likelihood) found via Newton's method using the gradient (Eqn. 23).

# Model Of Human Behavior




Skill level achieved by a training policy

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

$-\infty \rightarrow +\infty$

**Chance-corrected beta binomial**

# Parameter fitting

**Model has a couple of free parameters**

- **how much variability in performance is there across individuals?**
- **how smooth is the function?**

**Free parameters fit to data via hierarchical Bayesian inference**

# Fact Learning Experiment

Associate each person with the name of their favorite sports team


Six training faces

30 seconds of training

Each face shown for duration *d* ms

→ each face shown 5000/d times

Immediate 2AFC testing following training

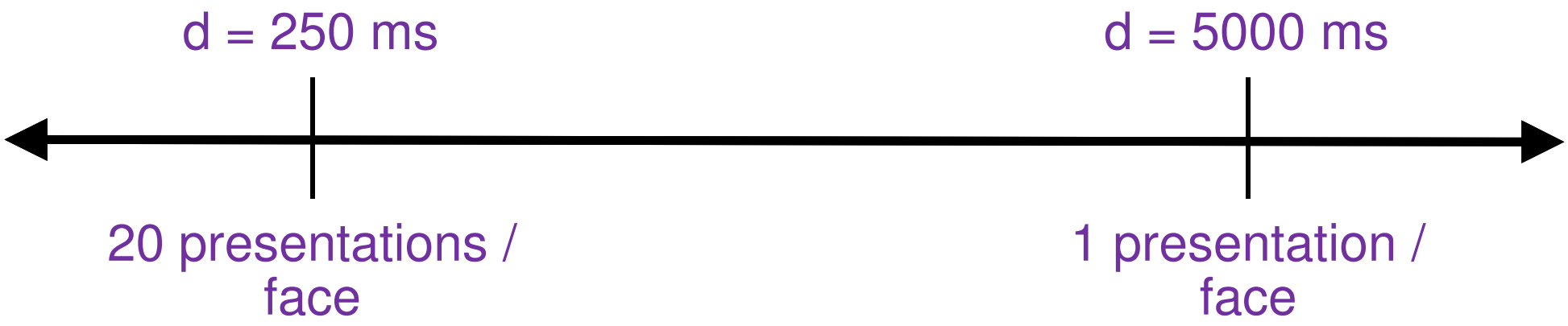
Demos

d = 250 ms

d = 5000 ms

# Fact Learning Experiment

## What is the optimal presentation duration?




**more presentations is better (with diminishing returns)**

**more time to process is better (with diminishing returns)**

## Trade off

# Fact Learning Experiment: Details

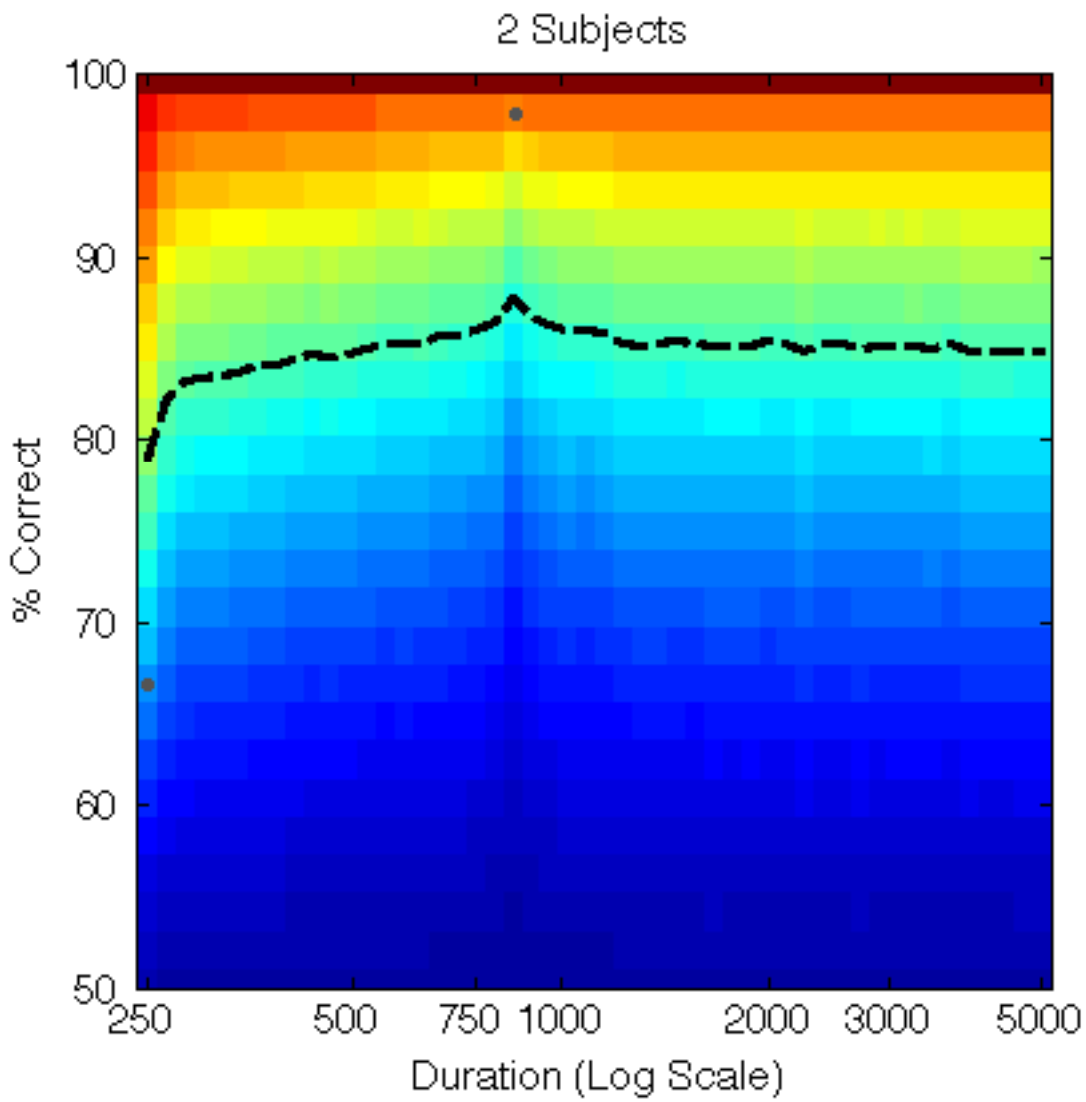
8 training/testing blocks with different faces

6 faces per block

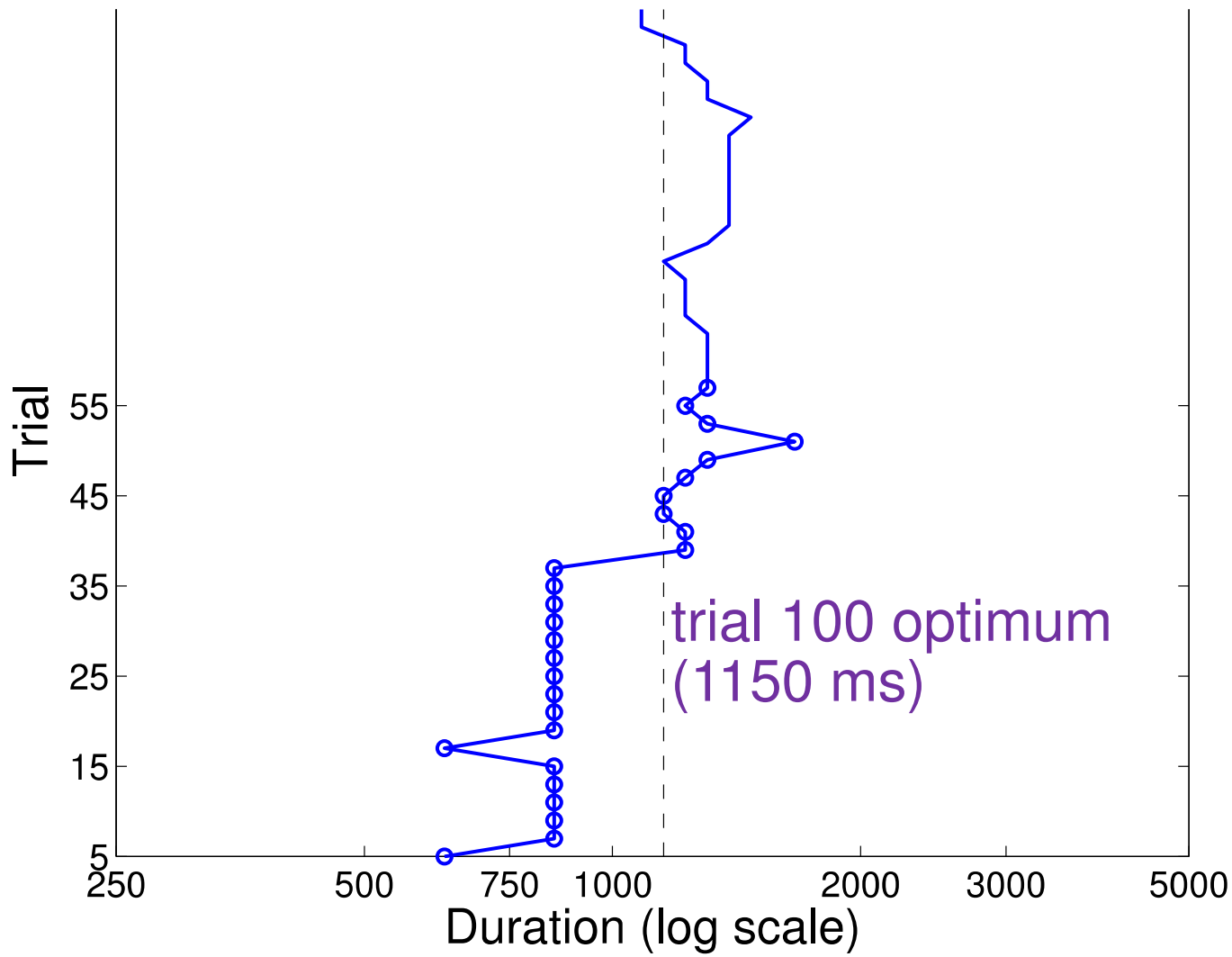
run on Mechanical Turk

30 cents/subject

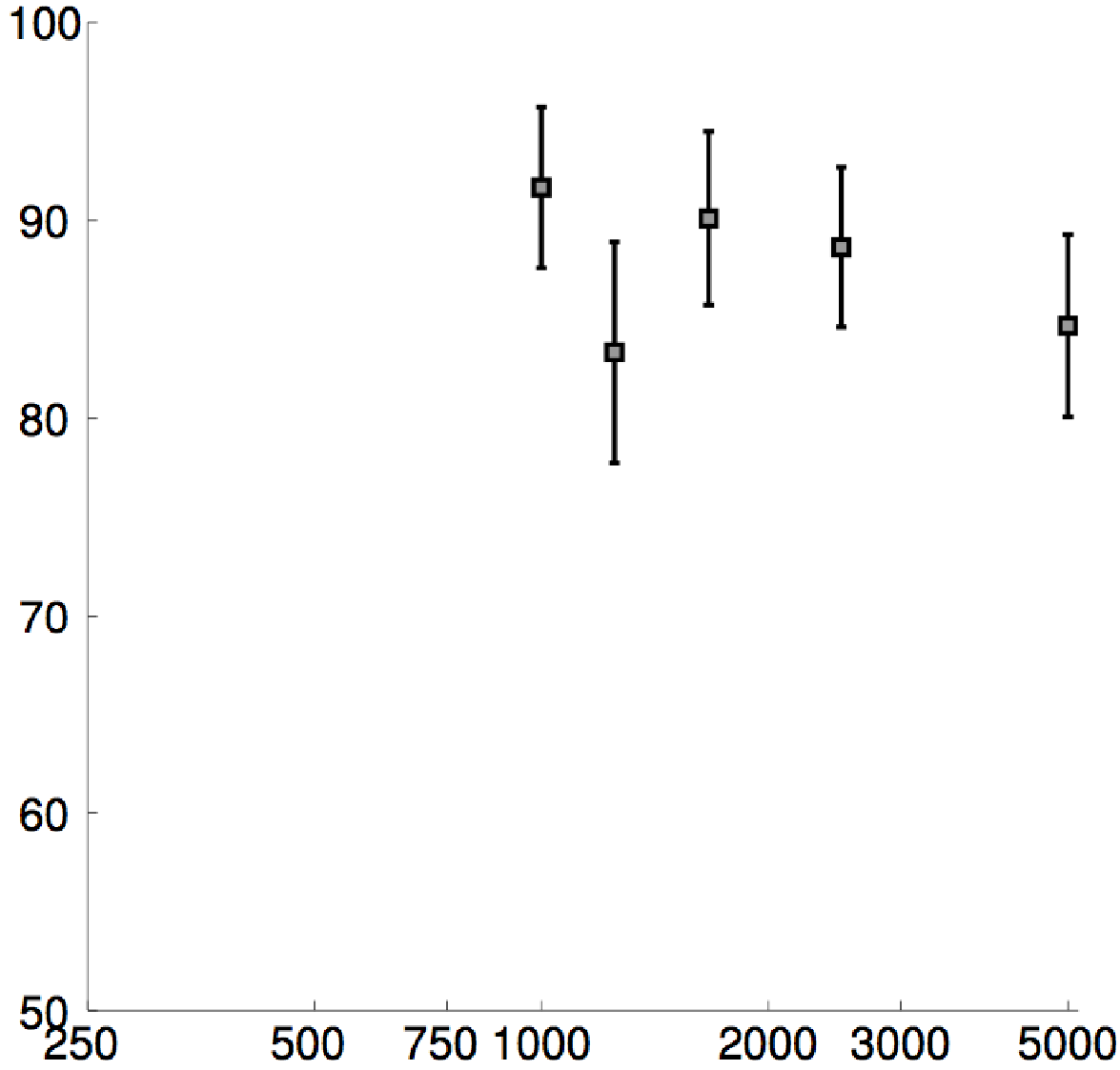
# Fact Learning Experiment: Optimization



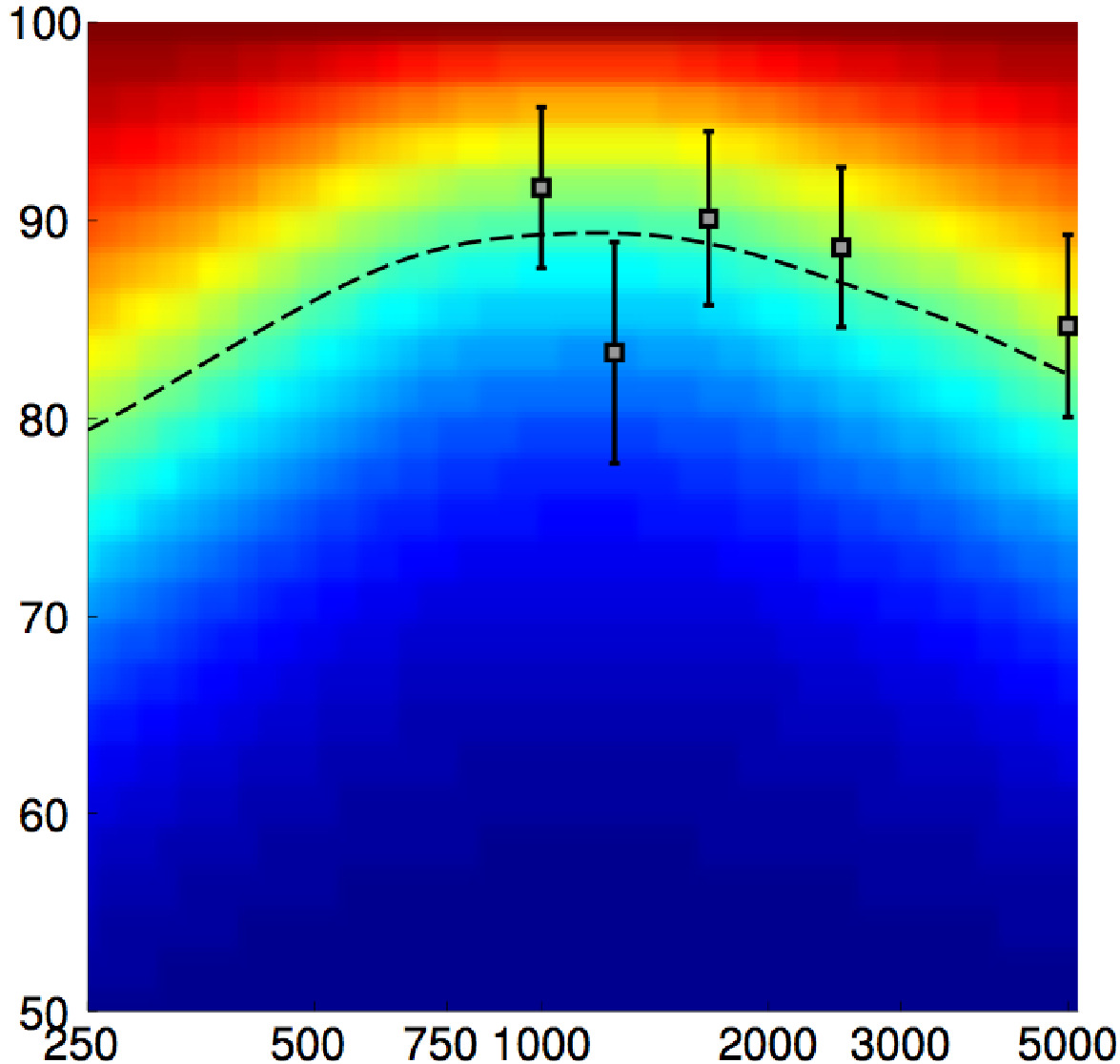
# Convergence



# Comparison With Traditional Experiment



# Comparison With Traditional Experiment



# Concept Learning Experiment

Amazon Mechanical Turk

Amazon Mechanical Turk

## Instructions

Imagine that you have encountered aliens from the Andromeda Galaxy who want to teach you their language. In the next few minutes, they will teach you the meaning of the GLOPNOR. GLOPNOR is a word that describes a set of objects. Some objects are GLOPNOR, other objects are not GLOPNOR. In the past, aliens have taught you words that mean 'breakable', 'bendable', 'larger than a toaster oven', and 'able to be used by two or more people at once.'

The aliens will show you a sequence of objects. For each object, you are to determine whether it is GLOPNOR or not. Initially, the aliens will give you feedback to tell you if your guess was correct. After these examples with feedback, the aliens will test your understanding of GLOPNOR by asking you to judge additional objects.

**You must complete the entire series of objects to receive payment. You can only participate once. This should take 5-10 minutes.**

Begin

Amazon Mechanical Turk

Amazon Mechanical Turk

https://workersandbox.mturk.com/mturk/accept?hitId=2XXL6VCPWZ9RTLA49QSSASN3JSMO9J&prevHitSubmitte

Google

Is this GLOPNOR?

No | Perhaps no | Don't know | Perhaps yes | Yes

Amazon Mechanical Turk

Amazon Mechanical Turk

https://workersandbox.mturk.com/mturk/accept?hitId=2XXL6VCPWZ9RTLA49QSSASN3JSMO9J&prevHitSubmitte

Google

Is this GLOPNOR?

No | Perhaps no | Don't know | Perhaps yes | Yes

Wrong! This is GLOPNOR.

Amazon Mechanical Turk

Amazon Mechanical Turk

https://workersandbox.mturk.com/mturk/accept?hitId=2XXL6VCPWZ9RTLA49QSSASN3JSMO9J&prevHitSubmitte

Google

(GLOPNOR)

Is this GLOPNOR?

No | Perhaps no | Don't know | Perhaps yes | Yes

Amazon Mechanical Turk

Amazon Mechanical Turk

https://workersandbox.mturk.com/mturk/accept?hitId=2XXL6VCPWZ9RTLA49QSSASN3JSMO9J&prevHitSubmitte

Google

(GLOPNOR)

Is this GLOPNOR?

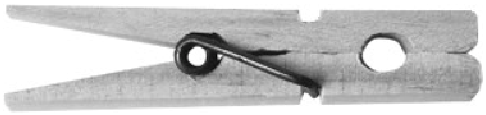
No | Perhaps no | Don't know | Perhaps yes | Yes

Correct! This is not GLOPNOR.

Amazon Mechanical Turk

Amazon Mechanical Turk

https://workersandbox.mturk.com/mturk/accept?hitId=2XXL6VCPWZ9RTLA49QSSASN3JSMO9J&prevHitSubmitte



(NOT GLOPNOR)

Is this GLOPNOR?

No | Perhaps no | Don't know | Perhaps yes | Yes

# GLOPNOR = Graspability

## Ease of picking up & manipulating object with one hand

## Based on norms from Salmon, McMullen, & Filliter (2010)

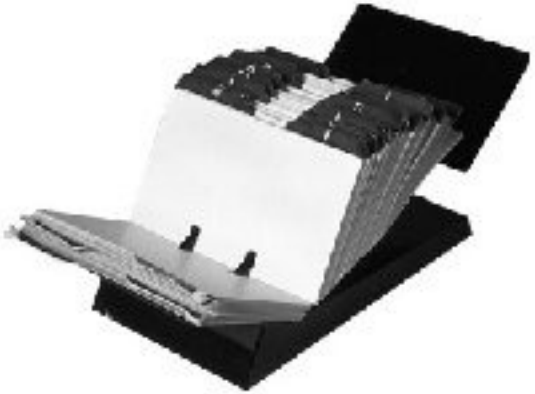
1.00

1.77

2.61

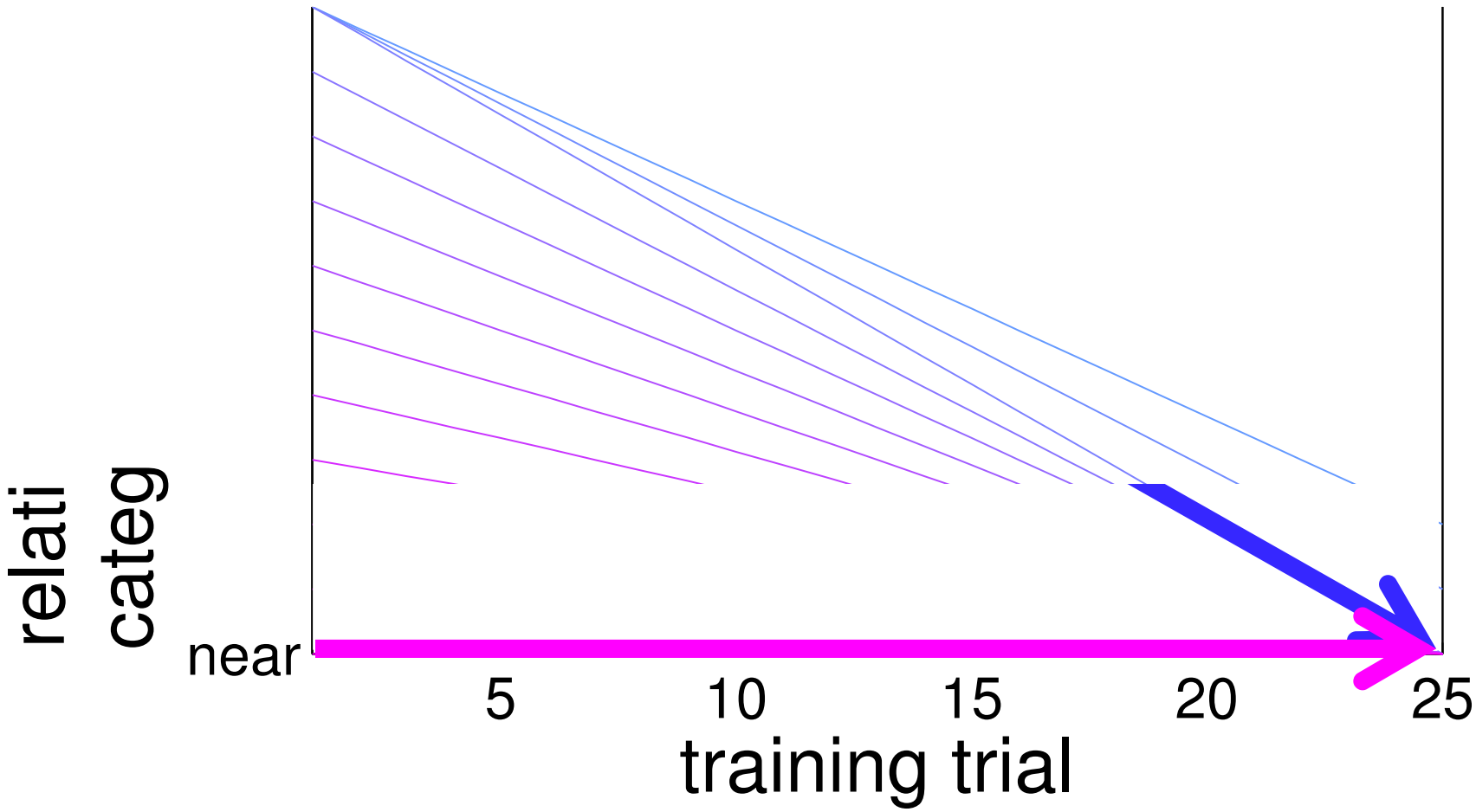
3.37

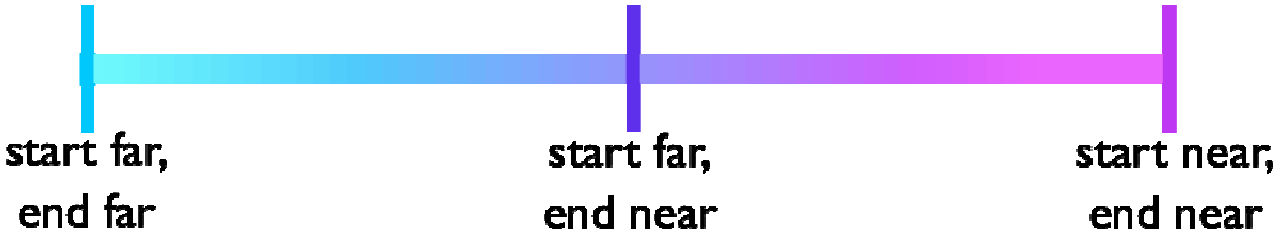
4.23



4.98

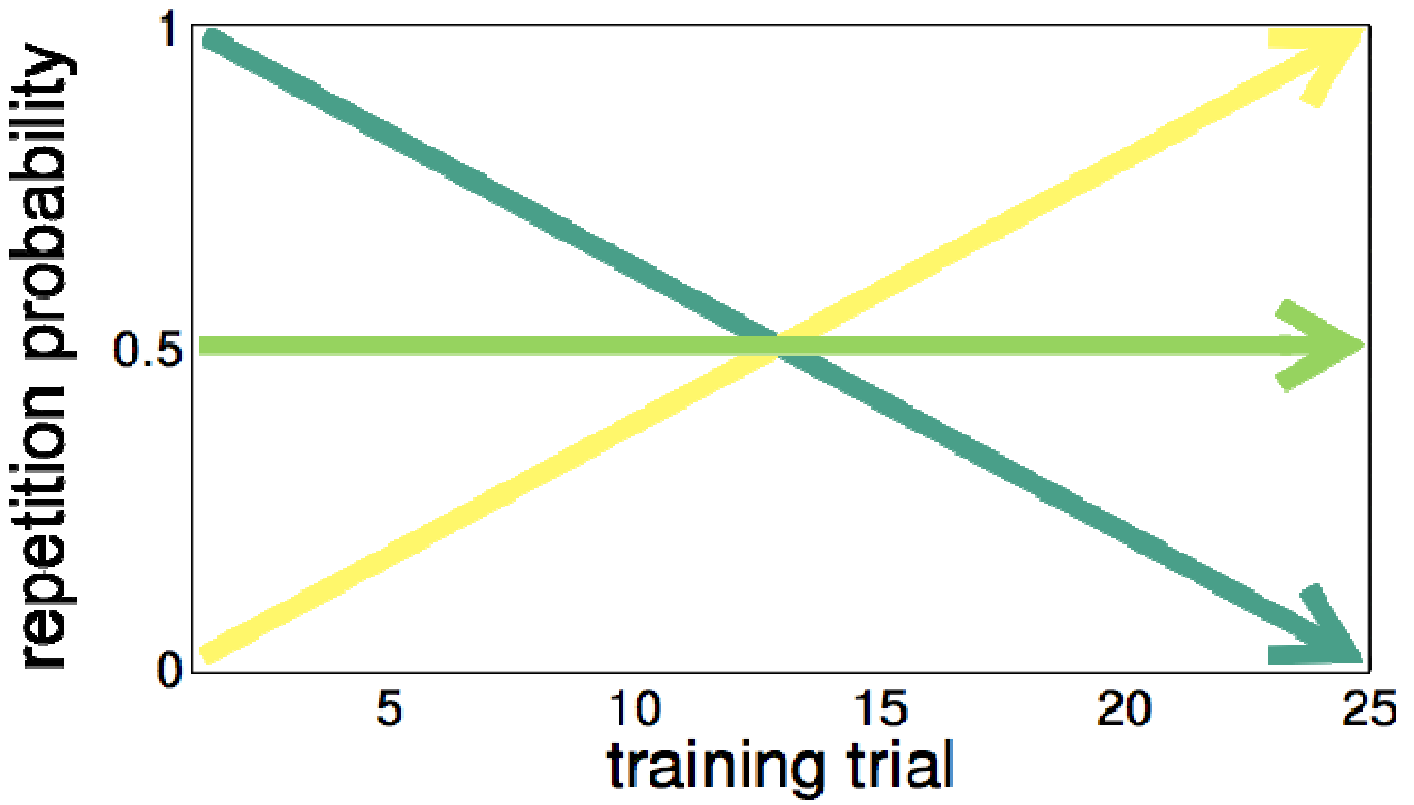
# Fading


relati
categ
near
5
10
15
20
25
training trial


start far,
end far
start far,
end near
start near,
end near

| Blocking | vs. | Interleaving |
| --- | --- | --- |
| + + + + − − − − | | + − + − + − + − |
| mostly repetitions | | mostly alternations |

# Blocking vs. Interleaving




**interleave early, block late**

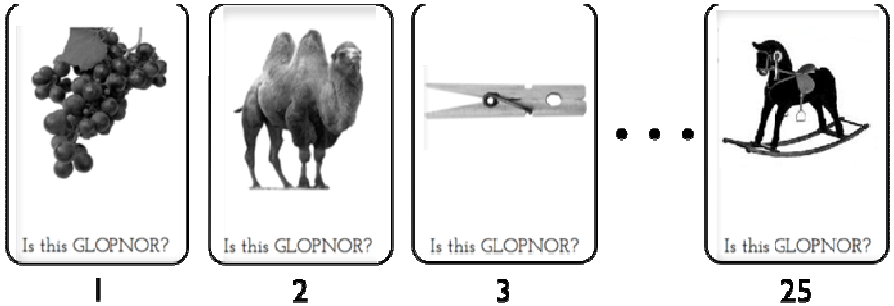
**no blocking or interleaving**

**block early, interleave late**

# Concept Learning Experiment

## Training

- 25 trial sequence generated by chosen policy



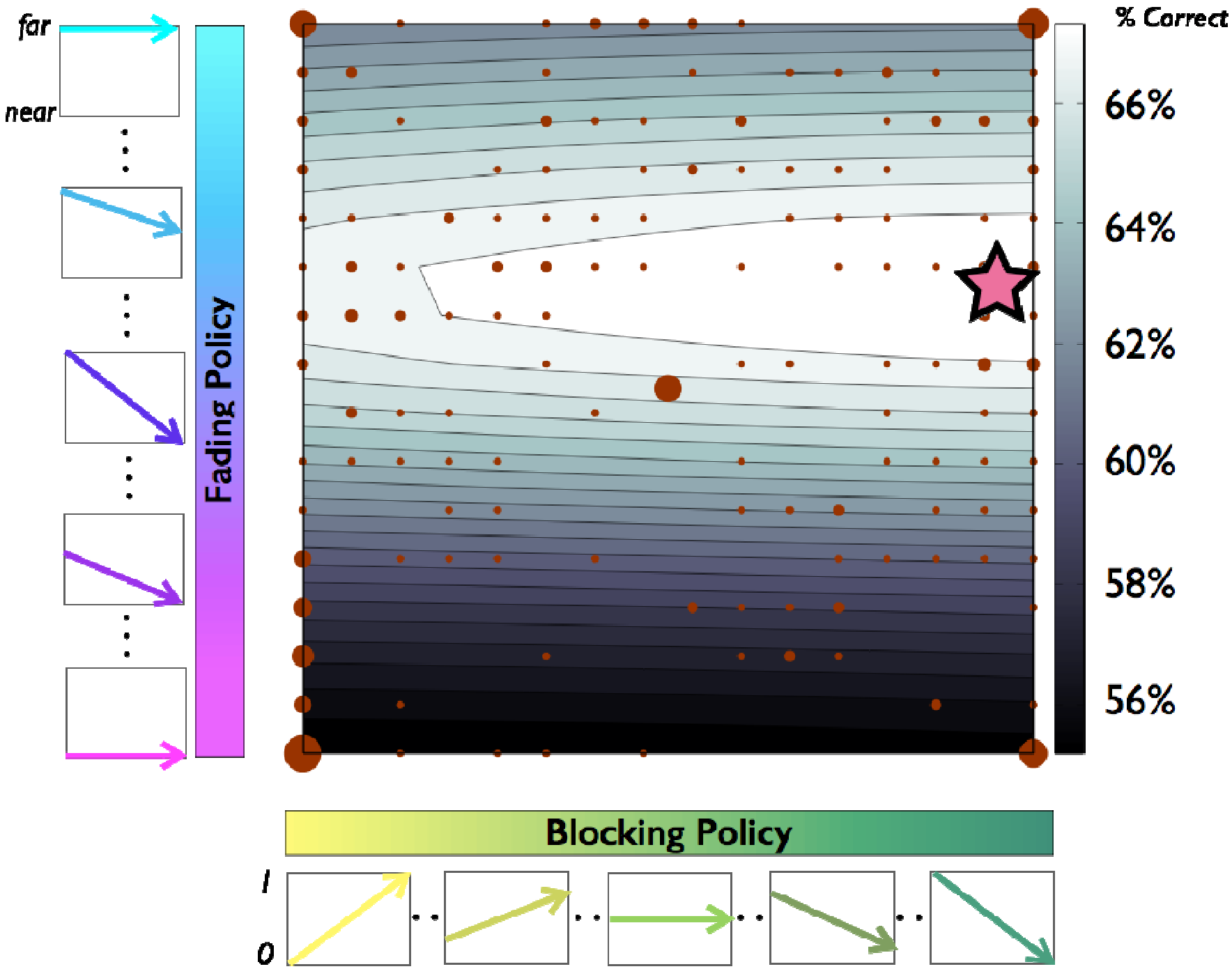

## Testing

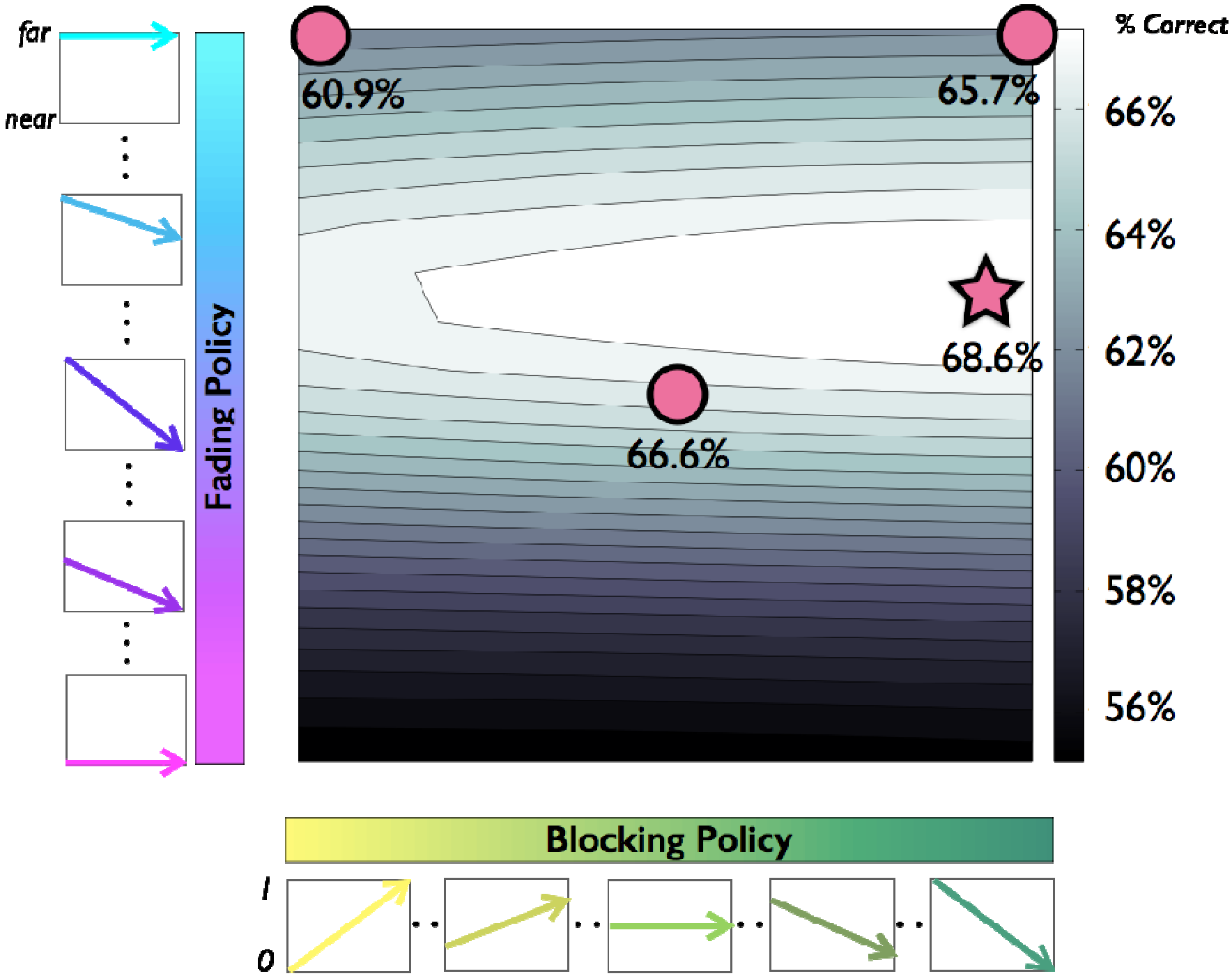
- 24 test trials, ordered randomly
- No feedback, forced choice

## Amazon Mechanical Turk

- $0.25 / subject

# Results


far
near
Fading Policy
% Correct
66%
64%
62%
60%
58%
56%
Blocking Policy
1
0


far
near
Fading Policy
60.9%
65.7%
68.6%
66.6%
% Correct
66%
64%
62%
60%
58%
56%
Blocking Policy
1
0

# Color Aesthetics

## Karen Schloss, Brown University

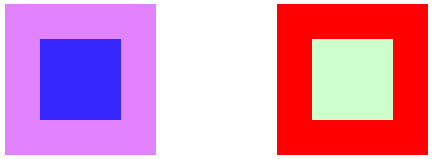
- the perception of color combinations
- how experience shapes preferences
- how preferences influence cognition and decision making

# Color Preferences

## Schloss and Palmer (2011)

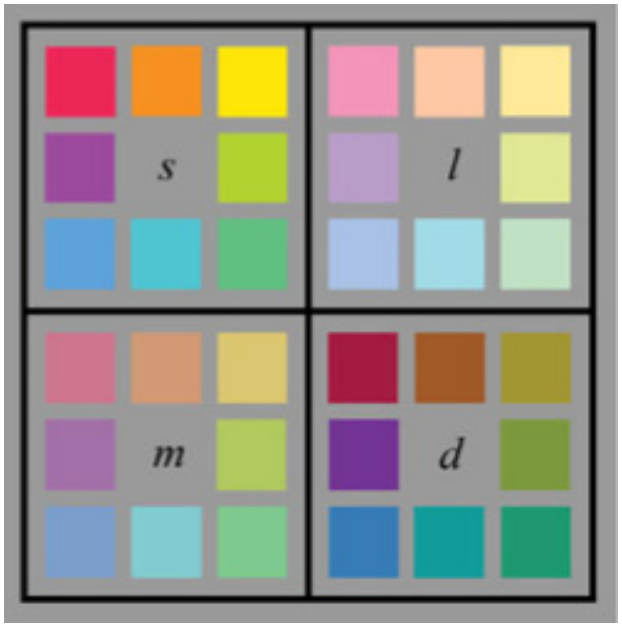
- present a wide variety of color pairs

  figure against a background

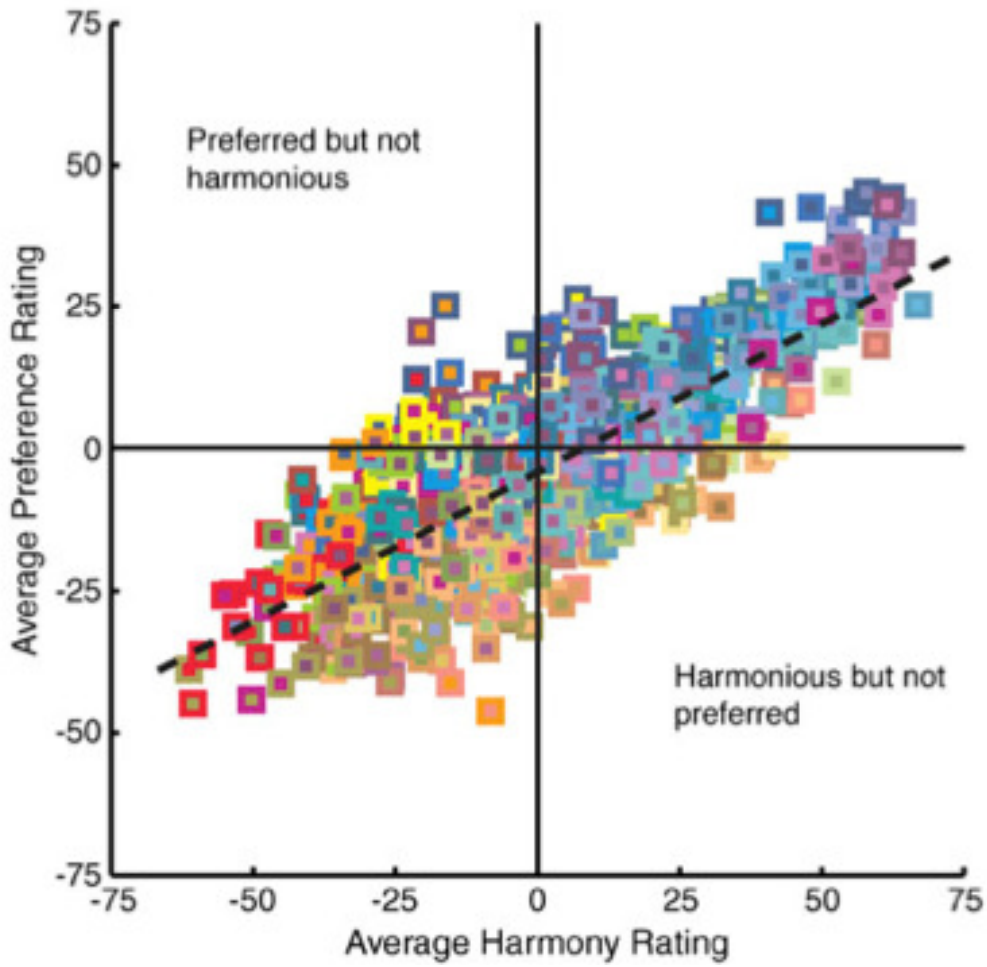


- asked 48 participants to rate how well the colors go together using a slider

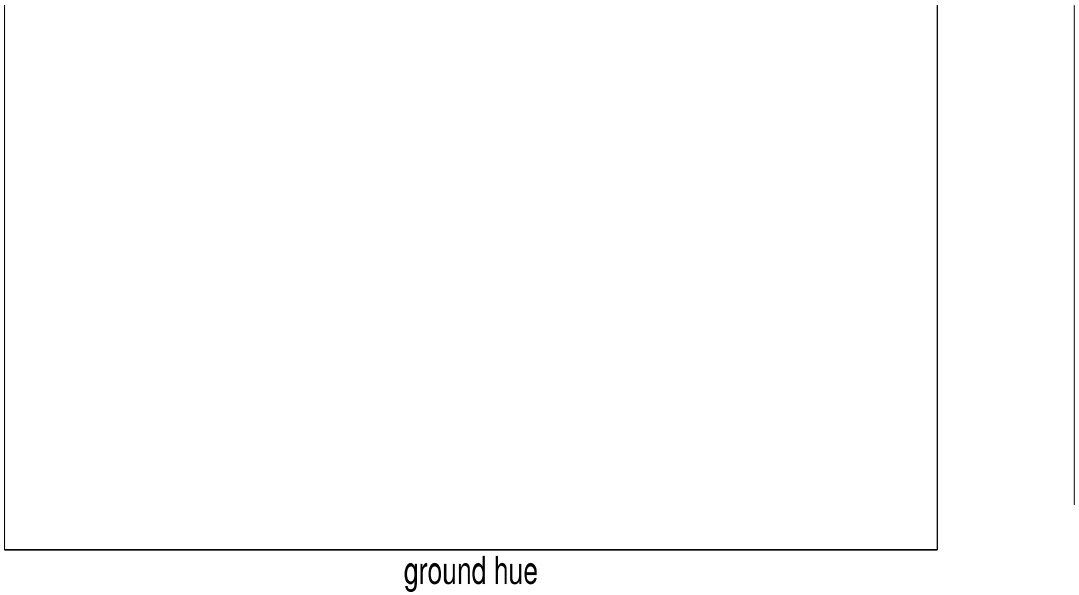
- 32 x 31 color pairs =
  992 ratings per participant



# Color Preferences



# Most And Least Preferred Combinations



# Charitable Giving

University of Colorado Foundation

GIVING TO CU NEWS & INFO ABOUT JOBS CONTACT

Home » Give Now

## Give Now

**MAKING A GIFT IS AS EASY AS 1, 2, 3**

1. Choose where you would like your gift to go
2. Enter the amount you would like to give
3. Add comments about this gift and add to cart

CU Faculty and Staff may give via payroll deduction. More information here.

### 1 CHOOSE WHERE YOU WOULD LIKE YOUR GIFT TO GO

Write-in the name of the fund you'd like to support here:

OR

Or browse for a fund within a campus:

- Anschutz Medical Campus »
- Boulder Campus »
- Colorado Springs Campus »
- Denver Campus »
- University of Colorado »

### 2 ENTER GIFT AMOUNT

would like to give:

- $
- $5000
- $1000
- $500
- $100
- $50

This is a recurring gift. (more info)

### 3 ADD TO CART

This gift is part of my pledged amount. (more info)

This is an honorary or memorial gift.

To make an honorary or memorial gift to a fund that is not a named honorary or memorial fund, please complete the forms below so we can contact the honoree or next of kin. If you are making a gift to a fund with the honoree's name in the fund title, this information is not necessary.

- In honor of (for a living person)
- In memory of (for a deceased person)

Add Comments on this Gift:

# Optimizing Donation Anchors

## Turk participants do a bogus task and get paid 5 cents.

## Then taken to donation page:

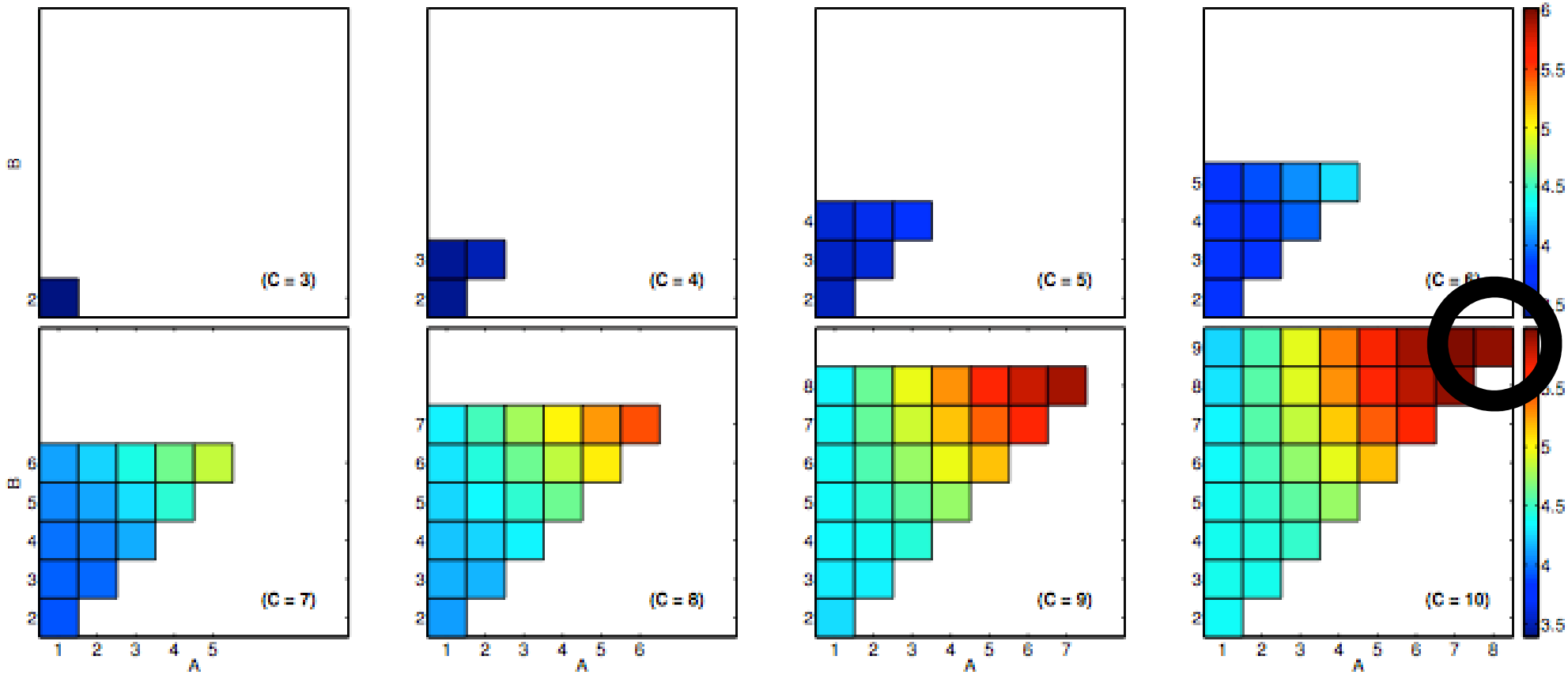
We will give you a 10 cent bonus. You may donate some or all of this bonus to the Red Cross for disaster relief. How much would you like to donate?

- ❑ 1 cent
- ❑ 3 cents
- ❑ 7 cents
- ❑ __ cents

# Optimizing Donation Anchors

Anchor triples: (A, B, C)

$A \in \{1, 2, \ldots, 10\}$
$B \in \{A+1, \ldots, 10\}$
$C \in \{B+1, \ldots, 10\}$




**Optimum at (8, 9, 10)**

# New Donation Experiment

- **Boring task for 20 trials**
- **Option to donate more time**

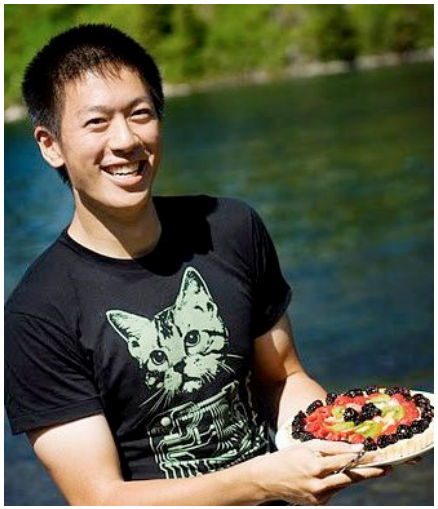
You've earned 5 cents now. We can't pay you any more, but for every additional 20 trials you pledge to do, we'll donate 1 cent to the Red Cross for disaster relief. If you do not complete your pledge, we will not donate.

How much would you like to donate?

- ❑ 1 cent
- ❑ 3 cents
- ❑ 7 cents
- ❑ __ cents

# Making Games Engaging

Yun-En Liu, University of Washington



## Treefrog Treasure

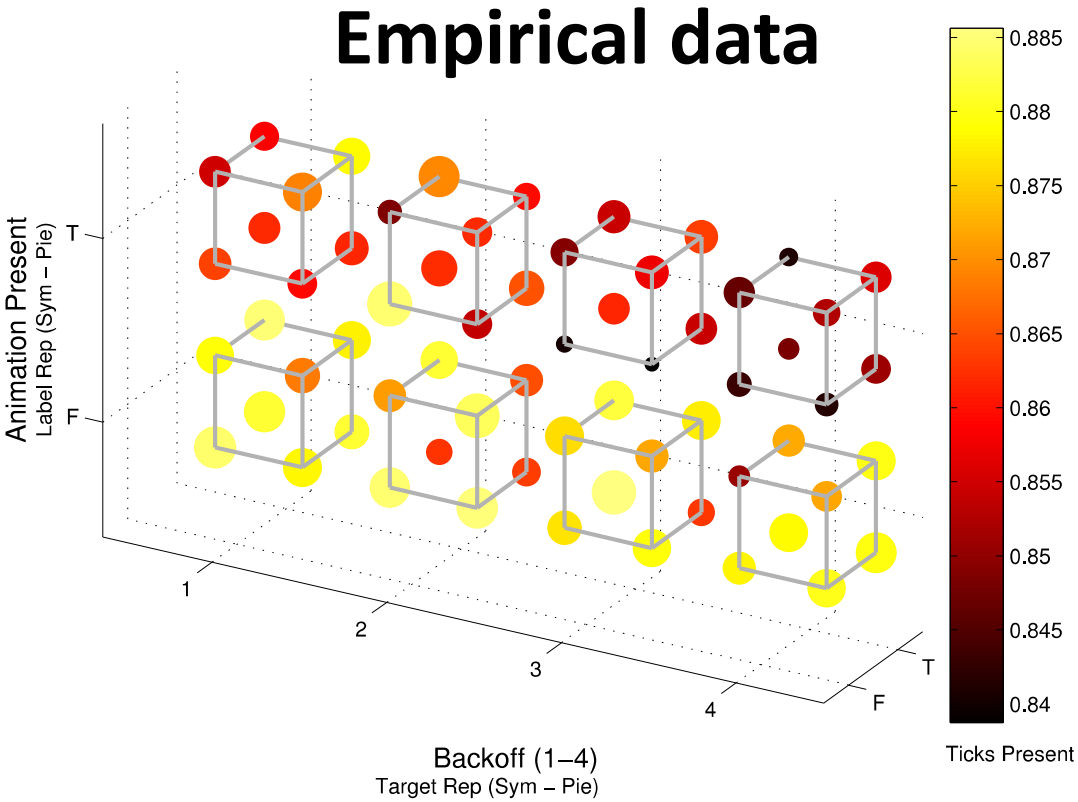
- educational game
- solve number line problems, learn fractions
- many variants of game

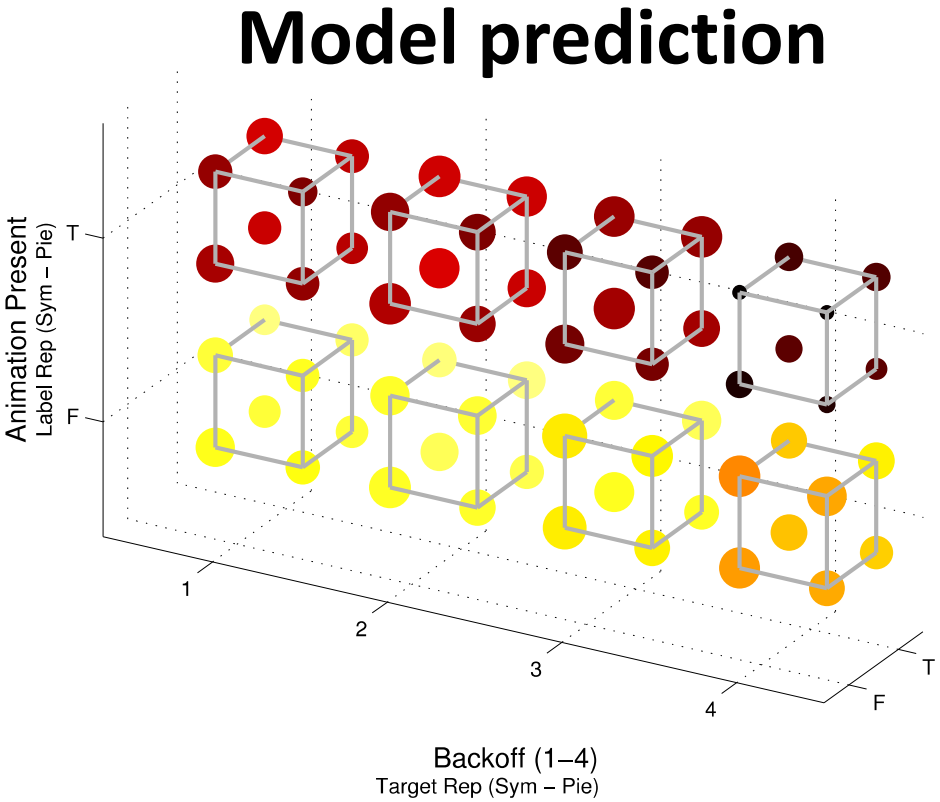
  2 x 2 x 2 x 4 configurations

# Making Games Engaging

## Which game configurations are more/less likely to cause student to quit playing?

- 360k trials, randomly assigned to 64 configurations



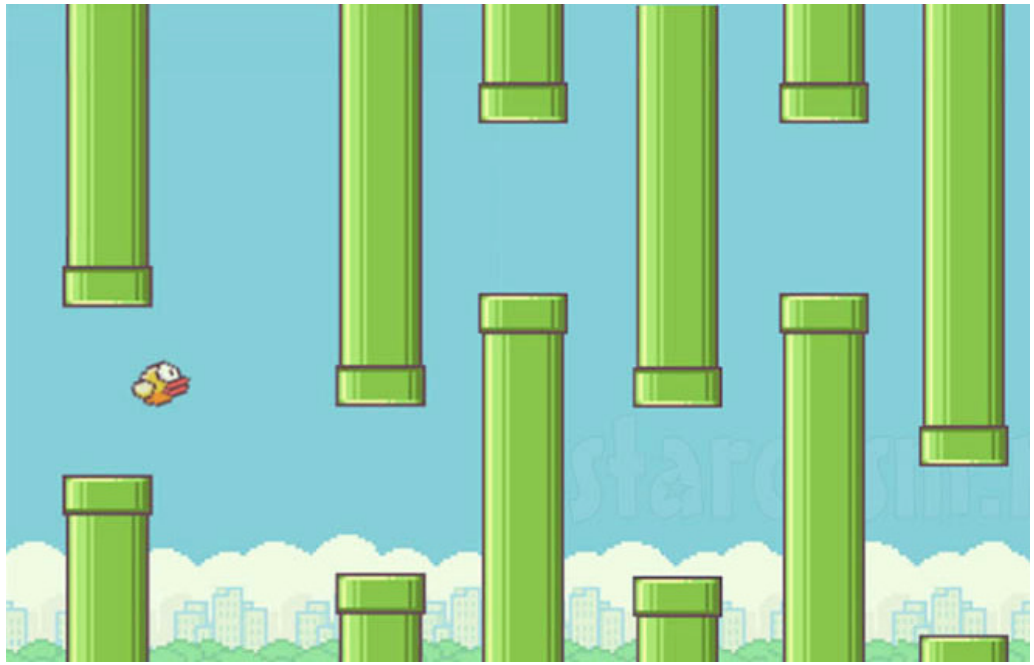



# Making Games Engaging II

Flappy bird

Many constants

- gap between pipes
- distance between pipes
- gravitational constant
- wing strength



Can we determine the optimum settings to make game more engaging for a novice?

# Bayesian Optimization: A-Z Testing

Alternative to traditional A/B testing

Allows us to efficiently search over a continuum of alternatives to discover an optimum

Machine learning techniques allow us to make stronger inferences from very noisy data.

Do we need this kind of smarts?

- Isn’t there an infinite supply of guinea pigs on the web?

# Why We Need Bayesian Optimization

## More efficient search leads to

- less bad press from running large experiments
- udent learning

THE WALL STREET JOURNAL.

- Furor Erupts Over Facebook's Experiment on Users

  Almost 700,000 Unwitting Subjects Had Their Feeds Altered to Gauge Effect on Emotion

- HUFF POST TECH ne to individuals, not populations

  **Why OKCupid's 'Experiments' Were Worse Than Facebook's**

# Thank you!

# Other Domains

**Determine optimal image transform to assist analysts and visually impaired**

Satgunam et al. (2012)

**Learn user-specific relationships**

- **e.g., Donation anchors as a function of # years since graduation**